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THALITA BRENNA DA SILVA MOREIRA

OUTPUT FEEDBACK FUZZY MODEL PREDICTIVE CONTROL APPLIED TO 3SSC BOOST CONVERTER

MOSSORÓ
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Dissertação apresentada ao Curso de Mestrado Acadêmico em Engenharia Elétrica do Programa de Pós-Graduação em Engenharia Elétrica da Pró-Reitoria de pesquisa e pós-graduação da Universidade Federal Rural do Semi-Árido, como requisito para obtenção do título de mestre em Engenharia Elétrica.

Área de Concentração: Sistemas de Controle e Automação

Orientador: Prof. Dr. Marcus Vinicius Silvério Costa

Coorientador: Prof. Dr. Fabrício Gonzalez Nogueira

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BANCA EXAMINADORA

MARCUS VINICIUS SILVERIO COSTA: 02599398380

Assinado digitalmente por MARCUS VINICIUS SILVERIO COSTA

Prof. Dr. Marcus Vinicius Silvério Costa

Prof. Dr. Fabricio Gonzalez Nogueira – UFC

VICTOR DE PAULA BRANDAO AGUIAR:99109573372

Assinado de forma digital por VICTOR DE PAULA BRANDAO AGUIAR:99109573372
Dados: 2020.12.15 16:38:57 -03'00'

Prof. Dr. Victor de Paula Brandão Aguiar

Prof. Dr. Bismark Claure Torrico - UFC

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Dedico este trabalho a minha querida avó Rita Almeida (in memorian)

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RESUMO

O recente avanço na capacidade computacional dos microprocessadores permitiu uma expansão nas pesquisas e aplicações diversas das técnicas de controle avançadas. Neste cenário, as técnicas de Controle Preditivo Baseado em Modelo (MPC – do inglês *Model Predictive Control*) e o controle *fuzzy* ganham destaque e popularidade devido as suas atrativas características. Esses métodos são capazes de tratar sistemas com restrições, incertezas no modelo, não-linearidades e perturbações externas. Dessa forma, considerando os bons atributos desses métodos, o objetivo deste trabalho é propor uma lei de controle que une as características dos controladores MPC com *fuzzy*. O método proposto consiste em um controle preditivo baseado em modelo *fuzzy* (FMPC– do inglês *fuzzy Model Predictive Control*) com realimentação de saída, ademais um modelo Takagi-Sugeno (TS) *fuzzy* e a técnica da compensação distribuída paralela (PDC– do inglês *Parallel-Distributed Compensation*) são usados para definição da lei de controle. Para projetar o controlador proposto, o FMPC com realimentação de estados é usado juntamente com um observador de estados *fuzzy*. Seguindo, critérios de estabilidade foram desenvolvidos de forma a garantir a estabilidade do sistema controlador-observador, considerando as abordagens *online* e *offline* do processo. Para realizar a análise do desempenho do controlador duas aplicações são executadas através de simulação computacional. Primeiro o controlador FMPC com realimentação de saída é aplicado a um exemplo numérico e depois a um conversor boost. Ademais, a análise é realizada para as metodologias *online* e *offline*, sendo a abordagem *online* comparada com controladores MPC com realimentação de saída encontrados na literatura. Os controladores são avaliados em termos da resposta no tempo, alocação de polos, índices de desempenho e elipsoides de estabilidade. Para ambas aplicações os resultados obtidos mostraram que o controlador proposto resolve os problemas de controle de forma eficiente, garantindo a estabilidade e desempenho do sistema mesmo diante de situações limitantes tais como: não-linearidades, mudança no ponto de operação, restrições de entrada e efeito de fase não-mínima.

Palavras-chave: Controle Preditivo Baseado em Modelo. Controle *fuzzy*. Modelos Takagi-Sugeno. Compensação distribuída paralela. Realimentação de saída. Critérios de estabilidade. Conversor boost.

ABSTRACT

The recent advance in the computational capacity of microprocessors has triggered an expansion of research and various applications of advanced control techniques. Considering this scenario, Model Predictive Control (MPC) and fuzzy control approaches gain prominence and popularity due to their attractive characteristics. These methods are capable of treating systems with constraints, uncertainties in the model, non-linearities and external disturbances. Thus, considering the good attributes of these control methods, the objective of this work is to propose and analyze a control law which merge the characteristics of MPC and fuzzy control. The proposed method consists of an output fuzzy model predictive control (FMPC), in addition a Takagi-Sugeno (TS) fuzzy model and the Parallel-Distributed Compensation (PDC) method is used to define the control law. In order to analyze the performance of the controller, two applications are run through computer simulation. First, the FMPC controller with output feedback is applied to a numerical example and then to a boost converter. Furthermore, the analysis is performed for the online and offline methodologies, with the online approach being compared with output feedback MPC found in the literature. The controllers are evaluated in terms of time response, pole allocation, performance indices and stability ellipsoids. For both applications the obtained results showed that the proposed controller solves the control problems efficiently, guaranteeing the stability and performance of the system even in the face of limiting situations such as: non-linearities, change in the operation point, input constraint and non-minimum phase.

Keywords: Model Predictive Control. Fuzzy Control. Takagi-Sugeno model. Parallel-Distributed Compensation. Output Feedback. Stability Criteria. Boost Converter.

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LIST OF ABBREVIATIONS AND ACRONYMS

3SSC	Three State Switching Cell
A-W	Anti-Windup
CCM	Continuous Conduction Mode
DCM	Discontinuous Conduction Mode
DMC	Dynamic Matrix control
FIR	Finite Impulse Response
FMPC	Fuzzy Model Predictive Control
GPC	Generalized Predictive Control
I/O	input and output
IAE	Integrated Absolute Error
ISE	Integral of Squared Error
ITAE	Integral of Time-weighted Absolute Error
ITSE	Integral of Time-weighted Squared Error
LPV	Linear Parameter Varying
LTV	Linear Time-varying
MF	Membership Function
MPC	Model Predictive Control
RMPC	Robust Model Predictive Control

CONTENTS

1 INTRODUCTION

This study deals with two advanced control techniques: the model predictive control and the fuzzy control, such methods have been gaining space in both academic and industrial space due to their viability to real applications. In view of this, this dissertation proposes a control approach which unites the aforementioned approaches, and also displays the viability of the proposed control system for two different applications.

Hence, before initiating the theory behind the studied procedures in the following chapters, the present chapter introduces the principal researches and background regarding MPC and fuzzy control in Section 1.1, another important tools, which are used to design the proposed controller, are also discussed. Furthermore, Section 1.2 brings together the most recent studies developed for the covered topics. Considering the discussion from Sections 1.1 and 1.2, the main proposal for this dissertation are listed and explained in Section 1.3 and the general and specific objectives are described in Section 1.4. Finally, Section 1.5 resumes the subjects addressed in the following chapters.

1.1 Background and significance of the study

Advances in theories of control and adequacy of the computational capacity of current microprocessors have allowed the development and application of powerful and sophisticated control strategies, applied to real plants (VAZQUEZ *et al.*, 2016). Among these advanced strategies, stand out the model predictive control (MPC) and the fuzzy control, since these control methods feature useful advantages for complex applications, such as, multivariable systems, nonlinear models, time varying and constrained controllers (MACHADO, 2007).

Concerning MPC theory, there is a well established theoretical knowledge with wide applications in the most diverse areas, however considering linear model, as affirm Espinosa *et al.* (2005). This advanced control technique was first developed and used in the industrial environment over 40 years ago, and since then has been gaining attention from both academic and industrial control community (AGUIRRE *et al.*, 2007b). According to Maciejowski (2002) MPC is the only advanced control method that showed a significant impact in industrial control processes. Since its first appearance, MPC has been the subject of many researches that provide its analysis, enhancement and development of new approaches (CAMACHO; BORDONS, 2007). Espinosa *et al.* (1999) states that this popularity is due to MPC high performance and few tuning parameters, which facilitates practical implementations.

There are several MPC control strategies, which are based on the system model to predict its future behavior. In addition, these control techniques aim to minimize a given cost function within a prediction horizon. Some examples are the Generalized Predictive Control (GPC), Robust Model Predictive Control (RMPC) and the Dynamic Matrix control (DMC). The main difference between MPC strategies lies in the adopted model and cost function, as define Camacho e Bordons (2007).

Among MPC's methods, the procedure developed by Kothare *et al.* (1996) highlights. This control law found many applications due to its capacity to guarantee the stability and performance, even when subject to system constraints, model uncertainties, multivariable process, disturbances, reference trajectory tracking and time delay. Furthermore, the study made by Kothare *et al.* (1996) is based on the linear matrix inequalities (LMI) methodology, which helps improving the controller application, since this technique is able to solve convex optimization problems in polynomial time, and can easily represent robust control theory.

Although MPC characteristics allow the resolution of several linear problems efficiently, there are still some drawbacks for this method as pointed by Yu-Geng *et al.* (2013). These difficulties are evidenced when MPC is proposed for controlling nonlinear systems, which results in a complex control structures, with high computational and time burden, as affirm Khairy *et al.* (2010). Furthermore, the gap still existing between the theoretical and practical aspects of MPC application can also be highlighted (ESPINOSA *et al.*, 1999). Oppositely, fuzzy controllers are well suitable for dealing with nonlinear models. According to Kovacic e Bogdan (2006), this aspect explains the increase in applications using this approach, as well as the necessity of controlling process with model uncertainties, and systems with undefined disturbances.

As with MPC, fuzzy logic is inserted in the control scenario for a long time, this theory is part of the artificial intelligence (AI) methods, which is used to mimic human knowledge and way of thinking in order to solve real problems with efficiency. The first fuzzy logic approach was introduced by Zadeh (1965) with the objective of offering a way to translate the human way of thinking using linguistic values, developing a new class of systems named fuzzy sets (MACHADO, 2003). Furthermore, the theory developed by Zadeh (1965) was only used to solve a control problem in the discussion addressed by Mamdani (1974), which applied a fuzzy algorithm to control a steam engine, proving the potential of such controllers.

Nowadays, fuzzy control applications are common and diverse, specially for modelling systems, as Espinosa *et al.* (1999) highlight. In this sense, the Takagi-Sugeno (TS) methodology (TAKAGI; SUGENO, 1985) offers a popular alternative to model systems using fuzzy theory, because of its proven feature as an universal approximator (TANAKA; WANG, 2001; SEIDI *et al.*, 2012). Furthermore, T–S fuzzy models unites the qualitative knowledge of the system, through its fuzzy rules, and the system quantitative knowledge given by the adopted models (FENG, 2018).

Usually, the Parallel-Distributed Compensation (PDC) scheme is used to design a controller for a TS modeled systems. For this procedure, a control law is described using the same structure as a T-S fuzzy model. This approach was first designed in Sugeno e Kang (1986) and then improved by Tanaka e Sugeno (1992), although it was only in Wang *et al.* (1996) that the PDC methodology was established and received its name.

Because of the ability of MPC to perform well when controlling systems with uncertainties and constrained, and the ability of fuzzy techniques to deal with non-linear systems, researches has been developed aggregating the characteristics of MPC and fuzzy control, in order to achieve control laws with greater ability to deal with real systems (ESPINOSA *et al.*, 1999). These approaches are called Fuzzy Model Predictive Control (FMPC). Considering the different types of MPC strategies, diverse FMPC controllers was proposed over the years, as in Hadjili *et al.* (1998), Huang *et al.* (2000) and Li *et al.* (2006).

Notwithstanding the solving characteristics for FMPC controllers, another limitation is often found for practical control applications: the need to measure all states of the system, which in practice is not always possible. An alternative to solve this problem is the use of a state observer to estimate the states of the system, thus featuring a control law with observer-based output feedback (PARK *et al.*, 2011). According to Kim *et al.* (2006), this type of strategy often presents difficulties in assuring the controlled system stability. Hence, the present studied follows the procedures developed by Wan e Kothare (2002), in which the state feedback controller and the state observer are designed individually, and then a stability criteria for their joint action is assessed.

Besides the aforementioned control strategies, this dissertation also deals with application for the studied methods. For this aspect of the proposed study, advanced control theories merge with the power electronic field. Following Costa (2017) this intersection is based on the need to use robust control methods to ensure the stability of power systems, even in the presence of usual disturbance, such as change of the operating point, constraints to the process, non-linearities and non-minimum phase. This need is also reinforced considering the current development of renewable energy system, which are constantly being subject of studies for improvements and are commonly integrated into power electronics structures.

Hence, the main purpose of this dissertation is to design a novel observer-based output feedback FMPC, considering the TS fuzzy model and PDC control law. Also, computational simulations are performed for a numerical example and a power electronic structure, in order to evaluate the viability of the method.

1.2 State-of-the-art

In order to complement the significance of the proposal and to understand the current scenario in which this dissertation is inserted, this section brings recent researches developed and the still existent gaps that this study aims to fulfill.

Since the MPC and RMPC theories are already consolidated among control researchers, there are a significant amount of recent researches, covering the most diverse areas, as can be seen in Oliveira *et al.* (2018) for a medical application and Chatrattanawet *et al.* (2017) with a fuel cells study. Furthermore, the applications given in Araújo e Coelho (2018), Shakeri *et al.* (2018), Hajizadeh *et al.* (2019), Velasquez *et al.* (2019), Cao *et al.* (2020) are also worth mentioning.

A similar outcome is found in the fuzzy control literature, including applications with T-S fuzzy modeling, such applications are found in studies as Liu *et al.* (2020a), which uses the T-S fuzzy method to design the non-linearities of an switched system, or Maroufi *et al.* (2020) with an approach involving wind energy. Besides, numerous others developments can be found, some of them are exposed in the following: Zhang *et al.* (2018), Hesamian *et al.* (2018), Ferrari *et al.* (2019), Cai *et al.* (2020).

Moving to the FMPC field, there are still many applications and developments, since exist diverse MPC and fuzzy methods there is a large field of action with studies in chemical engineering, as Teng *et al.* (2017), the transportation area e.g., Wang *et al.* (2018) and Dong *et al.* (2020), among others. Besides the specific applications, researches dealing with the improvement of this control theory are found in Yeh *et al.* (2006), Killian *et al.* (2015), Kaheni e Yaghoobi (2020).

Furthermore, the output feedback problem is widely spread and well established in control theory, with developments as the ones from Gu *et al.* (2019), Hu e Ding (2019), Manzano *et al.*

(2019), Xu e Zhang (2020), Liu *et al.* (2020b). Nevertheless, when dealing with output feedback FMPC procedures the number of researches is reduced and some space for improvements and new applications are found. In this theme, some research can be cited, such as Tang *et al.* (2018), Ping e Pedrycz (2019), in both an output feedback FMPC procedure is found for a TS fuzzy model. However, some gaps can be pointed out, such as not using the PDC strategy and using a non fuzzy state observer.

Beyond the output feedback FMPC existing gaps, currently advanced control procedure still has limited applications for power structures, some development in this field are Narimani *et al.* (2015), Costa (2017), Biswas *et al.* (2020), Hou e Li (2020). The researches limitations are even clearer for FMPC with or without output feedback approaches, some applications are discussed in Bououden *et al.* (2012), Baždarić *et al.* (2017), Rego e Costa (2020). Note that none of the aforementioned researches proceed an output feedback FMPC applied to a power electronic structure.

Thus, this study is situated according to the current scenario, and proposes an approach to satisfy existing gaps for enhancements and application of the proposed method. With this in mind, the main proposals and objectives of this dissertation are dealt with below.

1.3 Dissertation Proposals

The procedure proposed in this research is to implement an observer-based output feedback fuzzy model predictive control and introduce two different applications for the proposed method. Thus, considering this scope, the main contributions of the dissertation are summarized as follows:

- Adaptation of the state feedback FMPC control law developed by Li *et al.* (2000), considering the RMPC cost function from Kothare *et al.* (1996), besides a TS fuzzy model and PDC control;
- Implementation of the fuzzy state observer proposed by Feng (2018), in order to perform an output feedback control;
- Introduction of two new stability criteria for the observer-controller joint action, one for the online procedure and the other for the offline approach. These criteria are developed considering a TS fuzzy system associated with the PDC control strategy and a fuzzy state observer. This procedure is based on the method found in Wan e Kothare (2002), which only contemplates a linear model under nominal conditions.

- Development of methodologies considering an online and an offline implementation of the overall method;
- Application of the proposed control procedures for the numerical example of Park *et al.* (2011). Furthermore, the simulations results for the online method are compared with two output feedback benchmark controllers from Kim e Lee (2017) and Rego (2019). For the offline strategy the comparison was made with the online method, with the purpose of evaluate the system viability. For both approaches the obtained results are presented and discussed based on time response, poles allocation, performance indexes and stability invariant ellipsoid;
- Similarly, an application for the Three State Switching Cell (3SSC) boost converter from Costa (2017) is addressed. In addition, a comparison is made between the proposed control and the output feedback relaxed MPC developed in Rego (2019), for the online approach, and the offline analysis follows the numerical example. Following, the performance parameters are the same as those mentioned for the Park *et al.* (2011) model.

1.4 Objectives

The objectives of this work are divided into general and specifics, as presented by subsections 1.4.1 and 1.4.2.

1.4.1 General objective

The general objective of this study is to propose a new output feedback FMPC control law, by merging a state feedback fuzzy model predictive control and a fuzzy state observer, and then proposing a new stability criteria that guarantee the overall system stability. Furthermore, two different applications are implemented with the purpose of evaluating the controller performance.

1.4.2 Specific objectives

The following specific objectives are set to achieve the general objective.

- Propose a state feedback FMPC controller, based on the work developed by Li *et al.* (2000) considering a TS fuzzy model, a PDC control law and the cost function given for the RMPC from Kothare *et al.* (1996).

- Associate the proposed FMPC with the fuzzy state observer from Feng (2018), thus forming an output feedback control.
- Development of new stability criteria for the controller-observer procedure, considering the online and offline approaches.
- Apply the proposed control law to a numerical example and analyze its performance in comparison with benchmark controllers, in order to assess the feasibility of the proposal both online and offline procedures are implemented.
- Apply the proposed control law to a power boost converter and analyze the ability of the controller in maintain the reference tracking even in the face of limitations such as, input constraint and time variation. As with the numerical example, the online and offline approaches are analysed.
- Discuss the main obtained results and propose improvements for future studies.

1.5 Chapters Summary

The rest of the work is organised in sever chapters, each one of them is summarized as follows.

- Chapter 2 introduces the theoretical aspects of fuzzy control theory, including basics definitions, the composition of fuzzy systems, the different types of fuzzy systems and finally the Takagi-Sugeno fuzzy modelling and the Parallel-Distributed Compensation control approach.
- Chapter 3 addresses the model predictive control theory and the RMPC basic concepts, also providing a description of the main mathematical tools used to design the LMI-based RMPC control, such as Schur complement, Lyapunov stability and polytopic uncertainties.
- Chapter 4 describes the main procedures used to define the control law, including the FMPC state feedback controller, the fuzzy state observer and the proposed stability criteria. Furthermore, theorems are established in order to summarize the proposed procedures online and offline.
- Chapter 5 proposes an numerical application for the output feedback FMPC. Besides, the obtained results of the online computer simulation are analyzed in comparison with the output feedback MPC control laws from Kim e Lee (2017) and Rego (2019). This chapter also includes an analysis of the offline procedure in comparison with the online approach.

- Chapter 6 presents and discusses the results obtained from the application of the proposed control law to a boost converter. With the online procedure analyse made in comparison with the output feedback MPC from Rego (2019), and the offline approach is evaluated in contrast with the aforementioned online procedure.
- Chapter 7 includes the main conclusions about the study,and proposals to be developed in future works.

2 FUZZY CONTROL

This chapter discusses the theoretical aspects of fuzzy control, starting with a introduction of basic concepts, such as fuzzy sets and membership functions in Section 2.1. Next, Section 2.2 presents the three main structures that compose a fuzzy system and Section 2.3 discusses commonly used configurations for these systems. Moreover, the Takagi-Sugeno fuzzy design is detailed along with the Parallel-Distributed Compensation procedure in Sections 2.4 and 2.5, respectively. Finally, the main contributions of the chapter are highlighted in Section 2.6.

2.1 Basic Definitions

The theories of fuzzy logic and fuzzy sets were introduced by Zadeh (1965) and Zadeh (1988), with the purpose of representing classes or sets that can not be express using the usual mathematical logic, for example, common human expressions and thinking as *"much grater than"* or *"very high"*.

A classical mathematical set presents values that belongs or not to it, therefore its boundaries are well-defined. Using the classical logic, a given set A can be defined through a Membership Function (MF) μ_A, as given in (2.1).

$$\mu_A = \begin{cases} 1 \;\; If \;\; x \in A \\ 0 \;\; If \;\; x \notin A \end{cases} \tag{2.1}$$

In contrast, a fuzzy set do not present a well-established boundary, but a gradual transition represented by a membership function, which allows the representation of linguistic expressions. Thus, a fuzzy set can be defined as follows: for a collection of objects (or universe of discourse) X with a generic element given by x, then a fuzzy set B in X is a set of ordered pairs, as defined by (2.2).

$$B = \{(x,\ \mu_B(x))|\ x \in X\} \tag{2.2}$$

where, $\mu_B(x)$ represents the membership function of x in B. This function defines a value, or membership degree, to every $x \in X$ and it can assume any value between 0 and 1.

According to Wang (1997), the membership functions for fuzzy sets are crisp mathematical function used to express a fuzzy description. However, the design of MFs are subjective, thus different MFs can be used to express the same fuzzy description (JANG *et al.*, 1997). Although

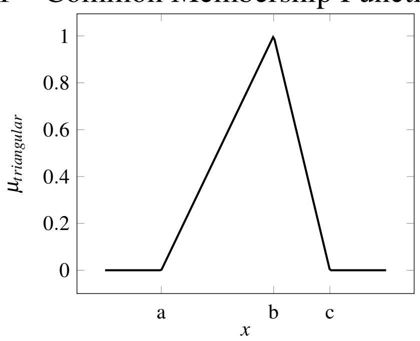
MFs can assume different forms, the most common are the triangular, trapezoidal, Gaussian and bell-shaped, which follow the expressions given in (2.3) and are illustrated in Figure 1.

$$
\mu_{triangular}(x) = \begin{cases} 0, \ \ for \ \ x < a \\ \dfrac{x-a}{b-a} \ \ for \ \ a \leq x < b \\ \dfrac{c-x}{c-b} \ \ for \ \ b \leq x < c \\ 0, \ \ for \ \ x > c \end{cases} \qquad \mu_{trapezoidal}(x) = \begin{cases} 0, \ \ for \ \ x < a \\ \dfrac{x-a}{b-a} \ \ for \ \ a \leq x < b \\ 1, \ \ for \ \ b \leq x < c \\ \dfrac{d-x}{d-c} \ \ for \ \ c \leq x < d \\ 0, \ \ for \ \ x > d \end{cases}
$$

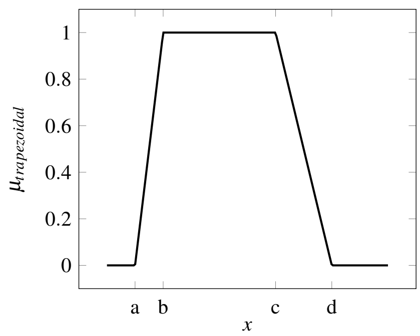
$$
\mu_{gaussian} = e^{-\left(\frac{x-c}{\sigma}\right)^2} \qquad \mu_{bell-shaped} = \frac{1}{1+\left|\frac{x-c}{\sigma}\right|^{2b}} \tag{2.3}
$$

for the triangular and trapezoidal MF the terms a, b, c and d are defined in Figure 1a and 1b. Moreover, for the Gaussian and bell-shaped membership functions c represents the center, σ the width and b controls the slopes at the crossover points (KOVACIC; BOGDAN, 2006).

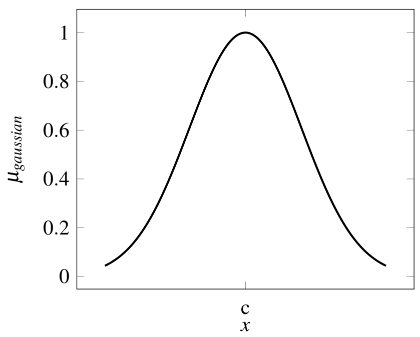
Figure 1 – Common Membership Functions formats



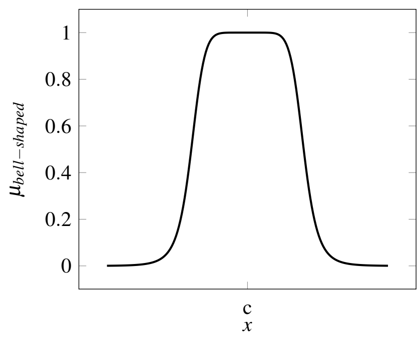
(a) Triangular Membership Function



(b) Trapezoidal Membership Function



(c) Gaussian Membership Function



(d) Bell-Shaped Membership Function

Source: The Author (2020)

Usually, the variables described by the MFs do not assume mathematical values, but words or expressions. Such variables are called linguistic variables and are necessary to represent the human knowledge on a subject. Take the speed of a car for example, which can be defined as a linguistic variable that can assume different linguistic values, such as "*slow*", "*average*" or "*very fast*" (KOVACIC; BOGDAN, 2006).

Following Kovacic e Bogdan (2006), a linguistic variable can be defined by (2.4).

$$[x, T, X, \mu] \tag{2.4}$$

where, x is the name (for the previous example: car speed), T represents the set of linguistic values that can assume (slow, average, very fast), X is the quantitative universe of discourse and μ the membership functions.

2.2 Fuzzy Systems

Fuzzy control has become one of the most popular and important topic for fuzzy researches, this happened due to fuzzy logic ability to convert human knowledge into a mathematical model (FENG, 2018; JANG *et al.*, 1997). According to Antão (2017), this strategy made possible to accurately represent real models and systems, forming the so-called fuzzy systems or fuzzy models.

Fuzzy systems are rule-based or knowledge-based systems that use fuzzy logic to represent the existent knowledge for a specific problem or to model the relation of the variables of a given system. The basic structure for these systems is composed of: knowledge base, inference engine, fuzzification and defuzzification interface, which are detailed in the following subsections (KACPRZYK; PEDRYCZ, 2015; FENG, 2018).

2.2.1 Knowledge base

The knowledge base is the foundation of a fuzzy model, and is formed by a rule base and a database. The latter gathers all membership functions, terms used to combine the rules, and linguist variables definitions (GEORGIEVA, 2016). As for the rule base, is constitute of a set of If-then rules and is the key part of a fuzzy system, in which all the others components are used to implement these rules efficiently (WANG, 1997; KACPRZYK; PEDRYCZ, 2015). The rule base is commonly express as a list of if-then rules, as shown in (2.5).

$$\begin{aligned}
&Rule\ (1): IF\ \ x_1\ \ is\ \ A_1^{(1)}\ \ and\ \ \ldots\ \ and\ \ x_n\ \ is\ \ A_n^{(1)}\ \ THEN\ \ \ y\ \ is\ \ B^1\\
&Rule\ (2): IF\ \ x_1\ \ is\ \ A_1^{(2)}\ \ and\ \ \ldots\ \ and\ \ x_n\ \ is\ \ A_n^{(2)}\ \ THEN\ \ \ y\ \ is\ \ B^2\\
&\vdots\\
&Rule\ (r): IF\ \ x_1\ \ is\ \ A_1^{(r)}\ \ and\ \ \ldots\ \ and\ \ x_n\ \ is\ \ A_n^{(r)}\ \ THEN\ \ \ y\ \ is\ \ B^r
\end{aligned} \tag{2.5}$$

where r represents the number of fuzzy rules in the rule base, A_i^l and B^l are linguistic values, $\mathbf{x} = (x_1,\ x_2,\ \ldots,\ x_n)^T$ and y are the input and output (I/O) linguistic variables, respectively.

Note that each If-Then rule can be divided into an antecedent part (If ...) and the consequent part (Then ...). Using traditional logic the if-then rule is only activate if x is exactly equal to A, then, the variable y is going to be precisely B. Whereas using fuzzy logic, the rule will be enabled when there is some degree of similarity between x (or premise variable) and A, as a result y will have some degree of similarity with B. The rules that form the rule base are necessary to represent human knowledge on a subject in order to achieve a solid fuzzy system. Moreover, the size of the fuzzy rule base depends on the number of input, output and linguistic variables that composes a system (MOZELLI, 2008; KOVACIC; BOGDAN, 2006).

The rule form of (2.5) is called canonical rule, and can include special cases of if-then rules, such as or rules, single fuzzy statement and gradual rules, which are represented as follows (WANG, 1997).

- Or Rules:

$$\begin{aligned}
&IF\ \ x_1\ \ is\ \ A_1^{(l)}\ \ and\ \ \ldots\ \ and\ \ x_m\ \ is\ \ A_m^{(1)}\\
&or\ \ x_{m+1}\ \ is\ \ A_{m+1}^{(l)}\ \ and\ \ \ldots\ \ and\ \ \ x_n\ \ is\ \ A_n^{(1)}\\
&THEN\ \ \ y\ \ is\ \ B^l
\end{aligned} \tag{2.6}$$

- Single fuzzy statement:

$$y\ \ is\ \ B^l \tag{2.7}$$

- Gradual rules:

$$The\ \ smaller\ \ the\ \ x,\ \ the\ \ bigger\ \ the\ \ y. \tag{2.8}$$

2.2.2 Fuzzy Inference System

The fuzzy inference system is responsible for interpreting the information defined on the knowledge base and produce an corresponding output. According to Mozelli (2008) and Passino *et al.* (1998), this procedure can be separated in four different steps:

1. Computing the compatibility degree of the premise variables with the rules antecedent, in (2.5) for example, would be defining the membership of x_n in the set $A_n^{(l)}$.
2. Defining the activation degree of a given rule. Mozelli (2008) states that this degree is obtained by combining the compatibility degrees from the first step. For the rules given in (2.5), each antecedent $(x_n \ is \ A_n^l)$ has a membership degree $\mu_{A_n^{(l)}}$ and the activation degree is given by the association of all membership degrees, following the logical connectives of the rule.
3. The two aforementioned steps match the input information with the premises of the if-then rules, and the third step procedures produce the corresponding output. The activation degree establishes the consequent result, considering the example, for a activation degree equal to 1 the consequent y is $B^{(l)}$.
4. The final procedure is called aggregation, which consists in combining the consequent of each rule, resulting in a fuzzy set or function.

2.2.3 Fuzzifiers and Defuzzifiers

The inference system processes a fuzzy set (input) resulting in another fuzzy set (output). However, most real applications use as input and output real values. Thus, it is necessary an interface between the environment and the inference system, which converts real values into fuzzy sets and vice-versa. These interfaces are known as fuzzifiers and defuzzifiers, respectively.

The fuzzification process can be described as a mapping from a real value $x \in X \subset R^n$ (input of the process) to a fuzzy set A in X (input of the inference system). Wang (1997) proposes three usual fuzzifiers: Singleton, Gaussian and Triangular fuzzifiers. The singleton fuzzifier reduce the computational demand for any type of membership function, however this fuzzifier cannot subdue input noise. On the other hand, Gaussian and Triangular fuzzifiers are capable of suppressing input noise, but are only computationally simplified for Gaussian and triangular membership functions.

Oppositely, the deffuzifier is a mapping from a fuzzy set B in Y (output of the inference system) to a real value $y \in Y \subset R$ (output of the process). There are three main deffuzification methods: center of gravity, center average and maximum deffuzifier. Among them, the center average type performs better, considering the three evaluation criteria: plausibility, computational simplicity and continuity (WANG, 1997).

2.3 Configurations for fuzzy systems

Although fuzzy systems are composed of the basic structures defined in Section 2.2, exist different associations for these components, forming various fuzzy systems. Following Wang (1997), three fuzzy systems can be highlighted and the main differences between them are the input and output variables. These systems are:

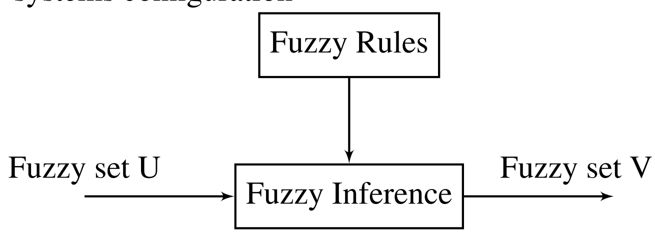
- Pure fuzzy systems;
- Mamdani fuzzy systems;
- Takagi-Sugeno fuzzy systems.

which will be described as follows.

2.3.1 Pure fuzzy systems

The pure fuzzy system presents the most basic fuzzy system configuration, which is composed of a fuzzy inference engine and the fuzzy rules (knowledge base). In these systems, both input and output are fuzzy sets, as illustrates Figure 2. According to Wang (1997), this feature jeopardize the performance for real applications, since usually the I/O are real value variables.

Figure 2 – Pure fuzzy systems configuration



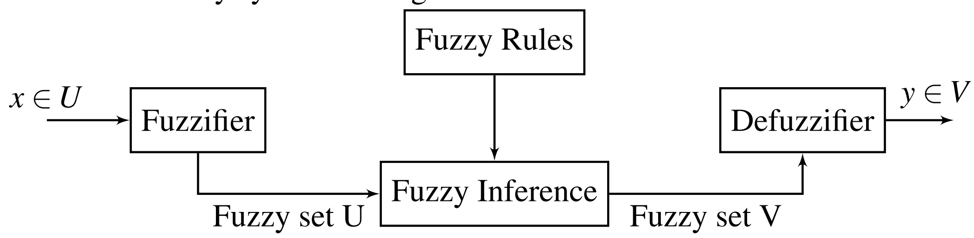
Source: Adapted from Wang (1997)

The scheme showed in Figure 2 can be used to explain the process of pure fuzzy systems. First, the fuzzy set $U \subset R^n$ feeds the fuzzy inference engine, which combines the group of if-then rules from the knowledge base, and then produces the fuzzy set $V \subset R$ as an output (WANG, 1997).

2.3.2 Mamdani fuzzy systems

The Mamdami fuzzy systems (or fuzzy systems with fuzzifier and defuzzifier) was proposed by Mamdani (1974) to overcome the main problem of pure fuzzy systems. Thus, for these systems although the process is made using fuzzy logic, the input and output are real values. Therefore, it is necessary a fuzzifier and a defuzzifier throughout the process, as showed in Figure 3.

Figure 3 – Mamdani fuzzy systems configuration



Source: Adapted from Wang (1997)

As illustrated by Figure 3, in the Mamdani configuration first a real value $x \in U$ goes through the fuzzifier, where it is turn into a fuzzy set $U \subset R^n$. Then, this set enters the fuzzy inference interface, which produces as output a fuzzy set $V \subset R$, considering the fuzzy rules base. The last step is to use a defuzzifier to turn the fuzzy set $V \subset R$ into a real value $y \in V$.

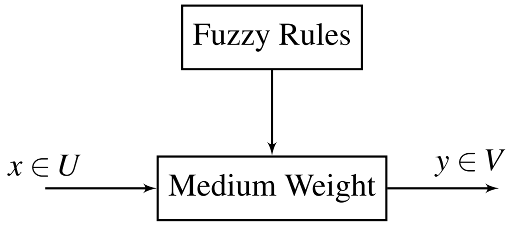
2.3.3 Takagi-Sugeno fuzzy systems

As with the Mamdami configuration, Takagi-Sugeno (TS) fuzzy systems also presents real values variables as input and output. However, for the TS systems the if-then rule are represented as in (2.9) instead of (2.5), with the consequent as a function of the input values and not a linguistic variable. This feature makes the TS model more suitable for use in engineering applications (MACHADO, 2003; BARROS *et al.*, 2016).

$$\begin{aligned} &Rule\ (i): \\ &If\ \ x_1\ \ is\ \ A_1^{(i)}\ \ and\ \ \ldots\ \ and\ \ x_n\ \ is\ \ A_n^{(i)} \\ &Then\ \ \ y^i = c_1^{(i)} x_1 + \cdots + c_n^{(i)} x_n \end{aligned} \tag{2.9}$$

The TS fuzzy configuration is given in Figure 4. The process begins with a real value $x \in U$ that enters the medium weight interface, which is obtained from the TS knowledge base, and results in the real value $y \in V$. Furthermore, the TS mathematical process that unites the rules base and the medium weight and converts an real value into other real value using fuzzy logic is detailed in Section 2.4.

Figure 4 – Takagi-Sugeno fuzzy systems configuration



Source: Adapted from Wang (1997)

2.4 Takagi-Sugeno Mathematical Model

The Takagi-Sugeno fuzzy method has been widely applied to model systems, since this method is more suitable for engineering application and feature as an universal approximator. Moreover, these systems are able to represent complex nonlinear models with uncertainties and disturbances using few if-then rules, and arrange nonlinear and linear control techniques (MOZELLI, 2008; FENG, 2018).

Following Seidi *et al.* (2012), a TS fuzzy model represents a nonlinear system through several locals linear input-output relations. Then, using fuzzy logic, these local subsystems are aggregated resulting in a global representation of the system.

For a given nonlinear system and considering a discrete fuzzy model, the T-S fuzzy representation is expressed by a set of If-Then rules as in (2.10).

$$\begin{aligned} &Rule\ j: \\ &If\ \ z_1(k) = \mu_{j1} \ldots\ \ and\ \ z_p(k) = \mu_{jp} \\ &Then \begin{cases} x(k+1) = A_j x(k) + B_j u(k) \\ y(k) = C_j x(k) + D_j u(k) \end{cases} \end{aligned} \tag{2.10}$$

where, $z_1(k), \ldots,\ z_p(k)$ are the premise variables, which may be functions of the state variables, μ_{jl} are fuzzy sets representing the membership degree, with $j = 1, 2, \ldots, r$, and r equal to the number of fuzzy rules. In addition, the state vector is given by $x(k) \in R^n$, $u(k) \in R^m$ is the input vector and $y(k) \in R^q$ the output vector. $A_j \in R^{nxn}$, $B_j \in R^{nxm}$, $C_j \in R^{qxn}$ and $D_j \in \mathbb{R}^{qxm}$ are the state matrices of the local subsystems.

Besides, according to Tanaka e Wang (2001), the global output of the system is obtained by the fuzzy association of the linear subsystems and is expressed by (2.11).

$$\begin{cases} x(k+1) = \sum_{j=1}^{r} h_j(z(k))(A_j x(k) + B_j u(k)) \\ y(k) = \sum_{j=1}^{r} h_j(z(k))(C_j x(k) + D_j u(k)) \end{cases} \tag{2.11}$$

where $h_j(z(k))$ represents the weight of each rule and is given by (2.12).

$$h_j(z(k)) = \frac{w_j(z(k))}{\sum_{j=1}^{r} w_j(z(k))} \tag{2.12}$$

$w_j(z(k))$ is the activation degree of the j^{th} implication and is given as follows.

$$w_j(z(k)) = \prod_{l=1}^{p} \mu_{jl}(z(k)) \tag{2.13}$$

with, $\mu_{jl}(z(k))$ as the membership degree of $z(k)$ in the fuzzy set μ_{jl}.

Since the activation degree can be affirm as,

$$\sum_{j=1}^{r} w_j(z(k)) > 0,\ \ w_j(z(k)) \geq 0,\ \ j = 1, \ldots, r \tag{2.14}$$

thus,

$$\sum_{j=1}^{r} h_j(z(k)) = 1,\ \ h_j(z(k)) \geq 0,\ \ j = 1, \ldots, r \tag{2.15}$$

2.5 Parallel-Distributed Compensation

The Parallel-Distributed Compensation (PDC) approach was first addressed by Wang *et al.* (1995) and offers a controller design which uses the same structure as the TS fuzzy model discussed in Section 2.4. Therefore, in order to implement the PDC strategy, the nonlinear system must be modeled using the TS fuzzy procedure (2.10)-(2.15) (TANAKA; WANG, 2001; SEIDI *et al.*, 2012).

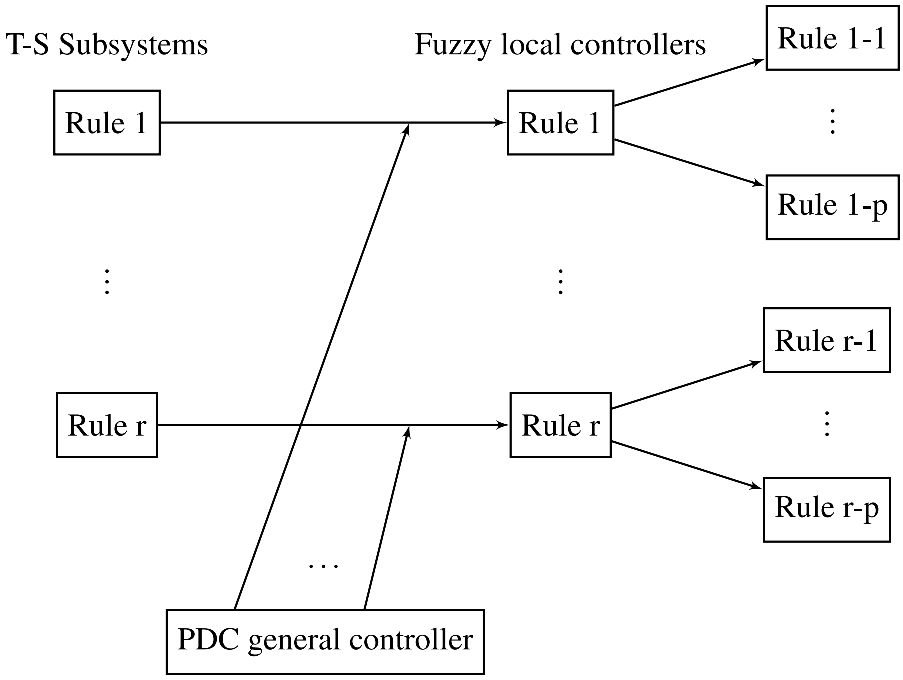
Mozelli (2008) defines PDC as a methodology in which a local controller is designed for each TS fuzzy if-then rule, sharing the same fuzzy set as the given model. These local controllers are then associated forming the control action, this procedure is illustrated in Figure 5. Mathematically, the PDC methodology is described in (2.16).

$$\begin{aligned} &Control\ Rule\ j: \\ &\quad If\ \ z_1(k) = \mu_{j1} \ldots\ \ and\ \ z_p(k) = \mu_{jp} \\ &\quad Then\ \ u(k) = -F_j x(k) \end{aligned} \tag{2.16}$$

For a given set of local controller described as (2.16), the PDC procedure performs a fuzzy association of these controllers obtained for each TS Fuzzy rule, resulting in the control law presented in (2.17).

$$u(k) = -\left(\sum_{j=1}^{r} h_j(z(k)) F_j\right) x(k) \tag{2.17}$$

Figure 5 – PDC Methodology



Source: Adapted from Seidi *et al.* (2012)

Replacing (2.17) in (2.11), the overall closed loop TS fuzzy system is given in (2.18).

$$\begin{cases} x(k+1) = A_z x(k) - B_z F_z x(k) \\ y(k) = C_z x(k) - D_z F_z x(k) \end{cases} \tag{2.18}$$

with, A_z, B_z, C_z and D_z representing the TS fuzzy state matrices obtained by the association of the local state matrices, as given in (2.19). In addition, the fuzzy combination for the gains from each rule results in the TS fuzzy gain F_z described in (2.20).

$$\begin{aligned} A_z &= \sum_{j=1}^{r} h_j(z(k))A_j, \quad B_z = \sum_{j=1}^{r} h_j(z(k))B_j, \\ C_z &= \sum_{j=1}^{r} h_j(z(k))C_j, \quad D_z = \sum_{j=1}^{r} h_j(z(k))D_j \end{aligned} \tag{2.19}$$

$$F_z = \sum_{j=1}^{r} h_j(z(k))F_j \tag{2.20}$$

Thence, the Parallel-Distributed Compensation technique has the purpose of finding the gains (F_j) which guarantee systems stability and control. This can be solved through LMI approach, by calculating the gain set that makes the system stable and also determining the Lyapunov matrix that assures the global stability of the closed loop system (MOZELLI, 2008).

2.6 Chapter's Summary

This chapter presented the theoretical aspects regarding fuzzy control methods, which are necessary to complement the fuzzy model predictive control methodology adopted in this study. To meet this objective, the basic definitions regarding fuzzy set and logic were introduced as well as the basic interfaces that compose a fuzzy control system. Moreover, the different possible configuration that a fuzzy system can assume was addressed, and a special consideration for the Takagi-Sugeno strategy was placed, since is the fuzzy model used throughout the dissertation. Lastly, the PDC procedure for controlling TS models was discussed and mathematically detailed.

3 MODEL PREDICTIVE CONTROL

The present chapter introduces the model predictive control theory by dealing with the basic definitions of MPC and the robust MPC strategy in Sections 3.1 and 3.2. Moreover mathematical tools commonly used for this control method are defined in Section 3.3, such as LMIs, Schur complement, Lyapunov criteria and polytopic uncertainties. Lastly, Section 3.4 resumes the main contributions of the chapter.

3.1 Basic Definitions

Model Predictive Control can be defined as a set of control strategies with shared characteristics, which are based on the prediction ability in a control process. Moreover, MPC strategies execute a control law that minimizes a certain cost function over a prediction horizon. In summary, the mpc strategy is defined by: (AGUIRRE *et al.*, 2007a; CAMACHO; BORDONS, 2007; WANG, 2009)

- Using an explicit model to predict future system output in a finite horizon. This model describes the dynamics of the system, including the possible disturbances and uncertainties;
- Calculating a control action which minimizes a given cost function;
- Performing an receding horizon control, for this strategy, although the future control is fully calculated, only the first control step is implemented. And then, the horizon is moved one step ahead.

Based on the aforementioned features, exist several MPC strategies, such as the Dynamic Matrix Control (DMC) developed by Cutler e Ramaker (1980), the Generalized Predictive Control (GPC) introduced in Clarke *et al.* (1987) and the Robust Model Predictive Control of Kothare *et al.* (1996). According to Aguirre *et al.* (2007a), the main differences between MPC methods are the adopted model, cost function and how to handle the control signal and constrains.

The essence of MPC is the model, which is responsible for represent the process allowing the obtainment of the predicted system output. For this reason, the model must properly represent the dynamic of the system as well as its uncertainties and disturbances (AGUIRRE *et al.*, 2007a). Different approaches can be used to design a model of a process, such as truncated impulse response model and transfer function model, however, the state space model (3.1) is the most usual for MPC applications (CAMACHO; BORDONS, 2007).

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{3.1}$$

where, the state variables is given by $x(t)$, the output voltage is given by $y(t)$, $u(t)$ represents the control signal, and A, B, C and D are state-space matrices.

Furthermore, the cost function (or objective function) depends on the performance objective of the control, such as, reducing the error between the reference and the output signal, and reducing the control effort. This function is implemented by the optimization process, in which is calculated the best result for the evaluated parameters. Usually, the cost function is represented by a quadratic function as given in (3.2) (WANG, 2009).

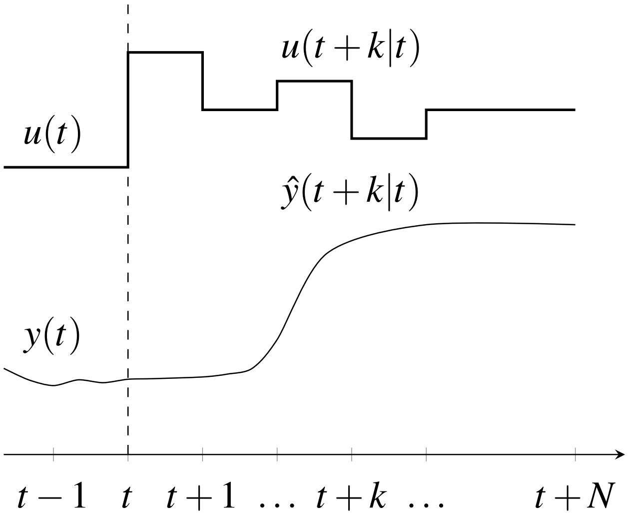
$$J = \underbrace{(R_s - y)^T (R_s - y)}_{\text{Error}} + \underbrace{u^T R u}_{\text{Control Effort}} \tag{3.2}$$

with, R_s as the reference vector and R is a weighting matrix used for tuning the control performance.

Following Camacho e Bordons (2007), all MPC methods share the same strategy, which is illustrated in Figure 6 and detailed as follows: First, at instant t, the future output $(\hat{y}(t+k|t))$ is predicted for the prediction horizon (N), considering the process model, the past control and output signals and the future control input $(u(t+k|t))$. This control signal is calculated through an optimization problem, which usually is a quadratic function of the output errors and control effort. Moreover, for the moving receding horizon the control input $(u(t|t))$ is sent to the process and the next control signals are rejected, since the signal $y(t+1)$ is known and is used to update the control signal for the forthcoming step.

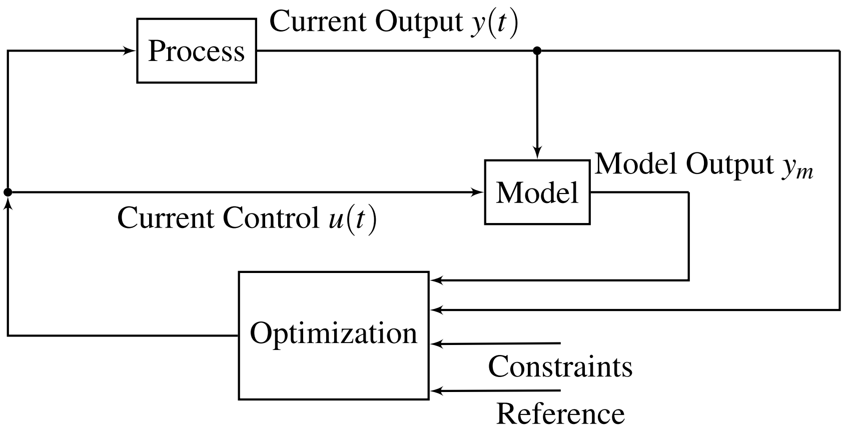
Following Camacho e Bordons (2007), Aguirre *et al.* (2007a), the basic structure for implementing the MPC strategy can be defined as illustrates Figure 7, which is explained as follows: the current control and output ($u(t)$ and $y(t)$) are used together with the model to define the predicted output of the system (y_m). This information is then used in a optimization process, which performs the minimization of an objective function, considering the reference and constrains of the system; and results in the new control step (AGUIRRE *et al.*, 2007a; CAMACHO; BORDONS, 2007).

Figure 6 – Reciding Horizon



Source: Adapted from Camacho e Bordons (2007)

Figure 7 – MPC basic configuration



Source: Adapted from Aguirre *et al.* (2007a)

3.2 Robust Model Predictive Control

A robust control is set to maintain a system stability and performance even in the face of uncertainties or disturbances. For these purpose a robust controller explicitly considers the differences between the real system and its model (CAMACHO; BORDONS, 2007; DAI *et al.*, 2012). According to Ogata e Yang (2002) a system designed through robust control theory has the following characteristic:

- Robust stability - the control system stays stable despite the presence of disturbances.
- Robust performance - specified control responses are found when the system faces disturbances.

The robust control theory can also be extended to predictive controllers, the RMPC strategy performs an optimization considering the worst scenario for the intrinsic uncertainties of the system. Among RMPC methods, Dai *et al.* (2012) highlights the min-max RMPC and the LMI-based RMPC methods, the latter being the one studied in this dissertation. More specifically, this dissertation is based on the LMI-based constrained RMPC developed by Kothare *et al.* (1996).

The method proposed by Kothare *et al.* (1996) is commonly used in MPC applications due to its capability to treat model uncertainty. This strategy consists of a *min-max* optimization problem, in which, at each sampling time, the predict future output of the system is calculated based on the process model. These predictions are then used to minimize the cost function $J_\infty(k)$, as given by (3.3).

$$\min_{u(k)} \ \max \ J_\infty(k) \tag{3.3}$$

with,

$$J_\infty(k) = \sum_{i=0}^{\infty}[x(k+i|k)^T W x(k+i|k) + u(k+i|k)^T R u(k+i|k)] \tag{3.4}$$

where, $W = W^T \geq 0$ and $R = R^T \geq 0$ are symmetric weighting matrices, $x(k+i|k)$ and $u(k+i|k)$ are the prediction steps ahead of the states and control, respectively. This optimization problem performs a search for the lowest control action considering the largest value of $J_\infty(k)$ (KOTHARE *et al.*, 1996).

The mathematical approach used by Kothare *et al.* (1996) for solving this min-max problem consists of: firstly deriving an upper bound on the cost function; And then, minimizing this bound with a constant state feedback control law as expressed by (3.5).

$$u(k+i|k) = Fx(k+i|k), \quad i \geq 0 \tag{3.5}$$

Hence, the quadratic function $V(x) = x^T P x$, with $P > 0$ is an upper bound on $J_\infty(k)$ if:

$$V(x(k+i+1|k)) - V(x(k+i|k)) \leq (x(k+i|k)^T W x(k+i|k) + u(k+i|k)^T R u(k+i|k)) \tag{3.6}$$

Besides, for the function (3.4) to be finite and the system perform robustly the states $x(\infty|k)$ must be null thus, $V(x(\infty|k)) = 0$. Thence, summing (3.6) from $i = 0$ to $i = \infty$ (3.7) is achieved.

$$V(x(k|k)) \geq J_\infty \tag{3.7}$$

Therefore, considering (3.3):

$$max\ \ J_{\infty}(k) \leq V(x(k|k)), \tag{3.8}$$

which represents the upper bound for (3.4). Moreover, following (3.8) it can be stated that the state feedback control law (3.5) assures the system stability at each sample time, considering the Lyapunov matrix P (KOTHARE *et al.*, 1996; SOUZA, 2015; COSTA, 2017).

3.3 Mathematical methods for RMPC

Some mathematical tools are necessary to well describe the LMI-based RMPC strategy, which are addressed in this section. First, the LMIs concept is defined in Section 3.3.1 and the Schur complement in Section 3.3.2. Moreover, Lyapunov stability criteria is discussed in Section 3.3.3 and Section 3.3.4 introduces the polytopic uncertainties design.

3.3.1 Linear Matrix Inequalities

The Linear Matrix Inequality (LMI) technique was first addressed in the control theory scenario over 100 years ago by Lyapunov. Since then, LMI methods have been developed and their use to address control problems has become popular. Such popularity can be explain by the possibility of using LMI methods to express robust control theory and solve optimization problems in polynomial time (KOTHARE *et al.*, 1996; CAPRON, 2014).

Boyd *et al.* (1994) define that for a variable $x \in \mathbb{R}^m$, a linear matrix inequality is express as in (3.9).

$$F(x) = x_1F_1 + x_2F_2 + x_3F_3 + \cdots + x_mF_m \geq -F_0 \tag{3.9}$$

or,

$$F(x) = F_0 + \sum_{i=1}^{m} x_iF_i \geq 0 \tag{3.10}$$

where $F_i = F_i^T \in \mathbb{R}^{nXn}$, $i = 0, \ldots m$, are the given symmetric matrices. Furthermore, $F(x)$ is positive semi-definite, that is, $u^T F(x)u \geq 0$ for all $u \neq 0 \in \mathbb{R}$. In short, a LMI can be defined as an inequality with matrix and symmetric elements (AGUIRRE *et al.*, 2007a).

Moreover, the LMI express in (3.9) is a convex constrain on x, i.e., its set solution given by $\{x \in \mathbb{R}^m | F(x) \geq 0\}$ is convex. This feature is very attractive, since allows the treatment of robust control problems with convex optimization, which minimizes an linear objective function of an given variable vector $x \in \mathbb{R}^m$ subject to a LMI constraint, such as the one given in (3.11) (AGUIRRE *et al.*, 2007a).

$$\begin{aligned} \min_{x} \quad & c^T x \\ subject\ to: \quad & F(x) \geq 0 \end{aligned} \tag{3.11}$$

According to Camacho e Bordons (2007), besides the convex optimization approach the LMI method can be used to solve problems as follows:

- Feasibility problem - finding the variables $x_1, x_2, \ldots, x_m$ that satisfy the inequality (3.9).
- Generalized eigenvalue minimization problem - calculating the minimum λ, satisfying $\lambda A(x) - B(x) > 0$, for $A(x) > 0$ and $B(x) > 0$.

These solving characteristics makes it possible to applied LMI in usual control theory constraints, such as Lyapunov stability criteria and convex quadratic matrix inequalities (BOYD *et al.*, 1994). In addition, nowadays an optimization problem formulated in terms of LMIs can be efficiently solved using algorithms known as LMIs solvers. Some of them are highlighted by Costa (2017), such as the Yalmip solver developed by Johan Lofbeg, the SeDuMi solver developed by Jos Sturm and the LMISol solver developed by Oliveira, Farias e Geromel in 1997.

3.3.2 Schur Complement

According to Costa (2017), Schur complement is a mathematical property commonly used to convert a convex inequality into a LMI or vice-versa. This conversion is made as explained as follows:

Given a matrix partitioned in four blocks as express in (3.12).

$$M(x) = \begin{bmatrix} M_1(x) & M_2(x) \\ M_2^T(x) & M_3(x) \end{bmatrix} \tag{3.12}$$

where, $M_1(x) = M_1^T(x)$, $M_2(x) = M_2^T(x)$ affinely depends on x and $M_3(x) = M_3^T(x)$ is a square and non-singular sub-matrix. Then, the Schur complement of M_3 in M_1, symbolized as (M_1/M_3) is defined as in (3.13).

$$(M_1/M_3) = M_1(x) - M_2^T(x) M_3(x)^{-1} M_2(x),\ for\ M_3(x) \geq 0 \tag{3.13}$$

In addition, if $M_1(x) \geq 0$ it is also valid to affirm that,

$$(M/M_1) = M_3(x) - M_2^T(x) M_1(x)^{-1} M_2(x),\ for\ M_1(x) \geq 0 \tag{3.14}$$

3.3.3 Lyapunov stability criterion

The stability is one of the most important aspect of control theory, and must be satisfy together with the controller performance to guarantee a control dynamic plausible to be applied in real systems. Hence, considering a given linear system (3.15), the stability is achieved when its derivative tends to zero, i.e., the system reach the stability when there is no more variation in the control system (AGUIRRE *et al.*, 2007a).

$$\Delta[x(t)] = Ax(t) \tag{3.15}$$

where, $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{nXn}$ and $\Delta[.]$ is a special notation that represents $\dot{x}(t)$ or $x(k+1)$ to continuous and discrete systems, respectively.

Thus, the equilibrium point x_e for the system (3.15) is given when $\Delta[x(t)] \equiv 0$, i.e., the trajectory stays permanently in x_e. Mathematically this indicates that (MOZELLI, 2008):

$$\begin{aligned} &\dot{x} = 0, \text{ for continuous time systems} \\ &x(k+1) = x(k), \text{ for discrete time systems} \end{aligned} \tag{3.16}$$

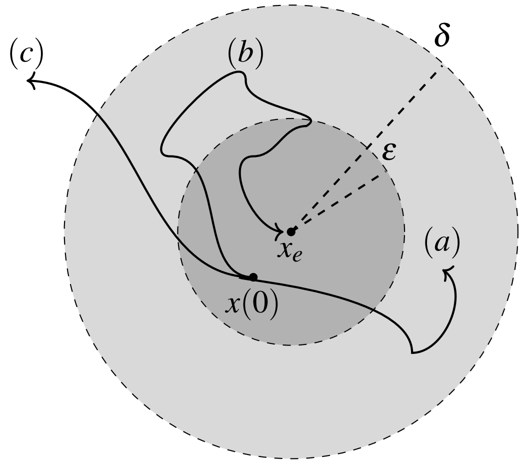
Moreover, Kovacic e Bogdan (2006) states that a system stability can be classified as:

- Stable - when small changes in the initial conditions generate small changes in state trajectory, thus:
  $\forall t_0,\ \forall \varepsilon > 0,\ \exists \delta :\ \|x(t_0) - x_e\| < \delta \Rightarrow\ \|x(t) - x_e\| < \varepsilon,\ t \geq t_0$
- Asymptotically Stable - when besides stable the system is attractive, i.e., trajectories that start nearby the equilibrium point converge to it:
  $\forall t_0,\ \exists \delta^* :\ \|x(t_0) - x_e\| < \delta^* \Rightarrow\ \lim_{t \to \infty} \|x(t) - x_e\| = 0$
- Globally Asymptotically Stable - when besides asymptotically stable δ^* is big enough.

Figure 8 illustrates a geometric projection for stable, asymptotically stable and unstable systems, considering $x \in \mathbb{R}^2$, a circle center in x_e with the initial conditions $(x(0))$ restricted within a radio ε and state trajectory confined in a radio δ.

Furthermore, according to Kovacic e Bogdan (2006) and Mozelli (2008), the stability methods based on Lyapunov theory is very widespread in the control theory literature, especially for fuzzy and MPC control. This method provides a mathematical resource used to search a equilibrium point for a system using a Lyapunov function for model representation, e.g., the polynomial Lyapunov function, the nonquadratic Lyapunov function and the quadratic Lyapunov function. Nevertheless, it is commonly used the quadratic function, which is detailed as follows.

Figure 8 – Projection for systems: (a) Stable, (b) Asymptotically Stable and (c) Unstable



Source: Adapted from Mozelli (2008)

For the linear system (3.15) and considering a quadratic positive-definite function, the Lyapunov stability criteria, is defined as shown in (3.17) (AGUIRRE *et al.*, 2007a).

$$V(x(t)) = x^T(t)Px(t) \geq 0, \quad with \quad P = P^T > 0, \; P \in \mathbb{R}^{nxn} \tag{3.17}$$

Besides, the derivative of (3.17) in the continuous time is given in (3.18).

$$\dot{V}(x(t)) = x^T(t)(A^TP + PA)x(t) \tag{3.18}$$

Since $V(x(t))$ is a positive-definite function, the equilibrium point is obtained if:

$$\dot{V}(x(t)) \leq 0 \tag{3.19}$$

or,

$$A^TP + PA \leq 0 \tag{3.20}$$

Thus, for a continuous time system, Lyapunov theorem assures that the system is asymptotically stable if there is a matrix $P = P^T > 0$ that satisfies the LMI in (3.20) (AGUIRRE *et al.*, 2007a).

Similarly, for a discrete time system the trajectory difference for the Lyapunov function (3.17) is represented in (3.21).

$$\Delta V(x(t)) = V(x(t+1)) - V(x(t)) = x^T(t)(A^TPA - P)x(t) \tag{3.21}$$

Therefore, Aguirre *et al.* (2007a) affirms that the discrete system is asymptotically stable if there is a matrix $P = P^T > 0$ and the inequality express in (3.22) is satisfied.

$$A^TPA - P \leq 0 \tag{3.22}$$

Following what was exposed above, the Lyapunov stability criteria consists of an optimization procedure that searches a matrix $P = P^T > 0$ that satisfies a given inequality, as express Aguirre *et al.* (2007a). Costa (2017) expands the Lyapunov stability casting the constraints using LMIs and Schur complement concepts:

Hence, for a discrete time system and considering a matrix $Q = Q^T \geq 0$, where $P = Q^{-1}$, (3.22) can be rewritten as (3.23).

$$A^T Q^{-1} A - Q^{-1} \leq 0 \tag{3.23}$$

Using mathematical artifice and rearranging (3.23) in the Schur complement form, (3.24) is obtained.

$$\begin{bmatrix} Q & QA^T \\ AQ & Q \end{bmatrix} \geq 0, \tag{3.24}$$

The expression (3.24) represents the LMI Lyapunov stability criteria, which can be described by the optimization problem (3.25)

$$\begin{cases} \min \; tr(Q) \\ \text{subject to :} \quad \begin{bmatrix} Q & QA^T \\ AQ & Q \end{bmatrix} \geq 0 \end{cases} \tag{3.25}$$

The following section explains the exposed concepts considering the polytopic uncertainty approach.

3.3.4 Polytopic uncertainty

When modelling real systems, it is common to appear differences between the model and the physical system, such differences are called system uncertainties. Since a faithful representation of the system is essential for its proper control, the treatment of uncertainties becomes an important part of robust control theory (GAHINET *et al.*, 1995).

As presented by Gahinet *et al.* (1995), usually these uncertainties appear when a simpler system is used as an approximation of a more complex system. Other causes of uncertainty are change in operating conditions, lack of knowledge of physical aspects of the system, varying time parameters and poorly designed models.

In RMPC theory, exist different techniques to compose a system uncertainty, such as the polytopic and the structured feedback uncertainty explored by Kothare *et al.* (1996). However, for the purpose of this study the uncertainty design is made through the polytopic method which is described as follows.

Aguirre *et al.* (2007a) defines that a polytope is a convex hull with a finite number of vertices, in which any set element can be obtained by the convex association of its vertices. That feature, coupled with Lyapunov stability theory, makes it possible to verify a uncertain system stability.

Mathematically, considering a system with n vertices as in (3.26):

$$\Delta[x(t)] = Ax(t), \quad A \in \mathbb{P} \triangleq \{A|A = \sum_{i=1}^{n} \alpha_i A_i,\ \alpha_i \geq 0,\ \sum_{i=1}^{n} \alpha_i = 1\} \tag{3.26}$$

The quadratic stability is achieved if there is a matrix $P = P^T \geq 0$ that makes valid the constrained (3.27)

$$\begin{aligned} &A^T P + PA \leq 0,\ \forall A \in \mathbb{P},\ \text{for continuous time systems} \\ &A^T PA - P \leq 0,\ \forall A \in \mathbb{P},\ \text{for discrete time systems} \end{aligned} \tag{3.27}$$

with the Lyapunov matrix P simultaneously satisfying all systems within the polytope.

Furthermore, to verify a system stability, it is not necessary to evaluate all infinite systems within the polytope, is sufficient to analyse its n vertices, as represented by (3.28) (AGUIRRE *et al.*, 2007a).

$$\begin{aligned} &A_i^T P + PA_i \leq 0,\ \forall i = 1,2,\ldots,n,\ \text{for continuous time systems} \\ &A_i^T PA_i - P,\ \forall i = 1,2,\ldots,n,\ \text{for discrete time systems} \end{aligned} \tag{3.28}$$

Kothare *et al.* (1996) used the polytope concept in order to represent the uncertainties of a state space Linear Time-varying (LTV) discrete model, this representation is given in (3.29).

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) + D(k)u(k) \end{cases} \tag{3.29}$$
$$[A(k)\ \ B(k)] \in \Omega$$

where, the set Ω is represented as a polytope:

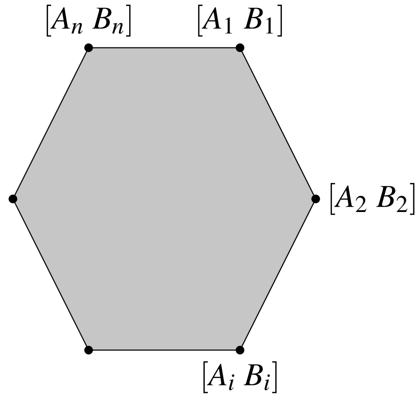
$$\Omega = C_o[A_1, B_1],\ [A_2, B_2],\ \ldots, [A_n, B_n] \tag{3.30}$$

with C_o representing a convex hull and its elements are given by the convex association of the vertices, as (3.31) (KOTHARE *et al.*, 1996).

$$[A,B] = \sum_{i=1}^{n} \lambda_i [A_i, B_i], \qquad \sum_{i=1}^{n} \lambda_i = 1, \lambda_i \geq 0 \tag{3.31}$$

Besides, an arbitrary geometric representation for the polytope (3.31) is displayed in Figure 9.

Figure 9 – Geometric projection for polytopic uncertainty



Source: Adapted from Kothare *et al.* (1996)

3.4 Chapter's Summary

This chapter presented the ground theory about model predictive control and distinguished the RMPC approach. These concepts are needed to design the proposed procedure. Furthermore, some mathematical techniques for the LMI-based RMPC was defined, such as the LMIs and Schur Complement concepts and their possibility in predictive control applications. Besides, the significance of dealing with system stability was highlighted and the Lyapunov stability criteria was discussed as a solution for these problems. Finally, the polytopic approach to design model uncertainties was addressed.

4 OUTPUT FEEDBACK FUZZY MODEL PREDICTIVE CONTROL

This chapter brings together the fuzzy and MPC theories, discussed in the previous chapters, and proposes a new control strategy. The developed procedure is an output feedback fuzzy model predictive control, and follows the methodology introduced by Wan e Kothare (2002), in which the output feedback characteristic is achieved combining a state feedback controller with a state observer, and then implementing a stability criteria for the controller-observer structure.

Hence, this chapter addresses the methods used to established the proposed controller. First, the state feedback control law and the state observer are introduced in Sections 4.1 and 4.2. Next, Section 4.3 explains the stability criteria for the controller-observer closed-loop model, and the overall proposed procedures are resumed for the online and offline approach in Sections 4.4 and 4.5. Finally, the main contributions of the chapter are listed in Section 4.6.

4.1 State Feedback Fuzzy MPC

This study is based on the state feedback FMPC controller developed by Li *et al.* (2000) which is stated as a LMI optimization problem such as (3.11). Moreover, the system stability is assured through Lyapuov functions considering a TS fuzzy model as (2.11) and the PDC control law as (2.17), with the uncertainties designed using the polytopic method.

Hence, the FMPC proposed in this dissertation is designed following the Li *et al.* (2000) procedure, however considering a state feedback strategy and presenting the cost function given in (4.1).

$$\underset{u(k)}{min}\ max\ J_\infty(k), \tag{4.1}$$

where,

$$J_\infty(k) = \sum_{i=0}^{\infty}[X(k+i)+U(k+i)], \tag{4.2}$$

with,

$$\begin{aligned} x(k+i) &= x(k+i|k)^T W x(k+i|k), \\ U(k+i) &= u(k+i|k)^T R u(k+i|k), \end{aligned} \tag{4.3}$$

$x(k) \in \mathbb{R}^{n_x}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the input signal or control action and $y(k) \in \mathbb{R}^{n_y}$ represents the output signal. Furthermore, $W = W^T \geq 0$ and $R = R^T > 0$ are weighting matrices, which are used to set up the controller performance.

Therefore, considering a TS fuzzy model as (2.11) and the PDC control law as (2.17), the state feedback FMPC strategy that solves the optimization problem (4.1) is given in Theorem 1.

Theorem 1 (Constrained Fuzzy Robust Model Predictive Control for TS fuzzy systems)**.**

$$\min_{\gamma,\, Q,\, Y_i} \gamma \tag{4.4}$$

Subject to the constraints given in (4.5)-(4.8).

$$\begin{bmatrix} 1 & x(k|k) \\ x(k|k)^T & Q \end{bmatrix} \geq 0 \tag{4.5}$$

$$\begin{bmatrix} Q & * & * & * \\ A_iQ+B_iY_i & Q & * & * \\ W^{\frac{1}{2}}Q & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y_i & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{4.6}$$

$$\begin{bmatrix} 4Q & * & * & * & * \\ S & Q & * & * & * \\ 2W^{\frac{1}{2}}Q & 0 & \gamma I & * & * \\ \sqrt{2}R^{\frac{1}{2}}Y_i & 0 & 0 & \gamma I & * \\ \sqrt{2}R^{\frac{1}{2}}Y_j & 0 & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{4.7}$$

$$\begin{bmatrix} Q & * \\ Y_i & u_{max}^2 I \end{bmatrix} > 0 \tag{4.8}$$

where, $S = A_iQ + B_iY_j + A_jQ + B_jY_i$.

Thus, the solution to this optimization problem results in obtaining the gain for the FMPC through the expression (4.9).

$$F_j = Y_jQ^{-1} \tag{4.9}$$

Proof. Following the procedure proven in Section 3.2, for the min-max problem given in (4.1), the Lyapunov function $V(x) = x^T(k)Px(k)$ is an upper bound of $J_\infty(k)$ if,

$$V(x(k|k)) \geq J_\infty \tag{4.10}$$

Therefore, the solution of (4.1) becomes sub-optimal, because the minimization is made considering the function $V(x(k|k))$ instead of the cost function J_∞. Thus, the optimization problem can be written as (4.11) (LI *et al.*, 2000).

$$\begin{aligned} &\text{minimize} \;\; \gamma \\ &\text{subject to} \;\; J_\infty < x^T(k|k)Px^T(k|k) < \gamma \end{aligned} \tag{4.11}$$

Taking $Q = \gamma P^{-1}$ and applying the Schur complement procedure,

$$\begin{aligned} &\text{minimize} \;\; \gamma \\ &\text{subject to} \;\; \begin{bmatrix} 1 & x(k|k) \\ x(k|k)^T & Q \end{bmatrix} > 0 \end{aligned} \tag{4.12}$$

Furthermore, for the TS fuzzy model and the PDC control law given in (2.11) and (2.17) the closed-loop system can be described as (4.13) or (4.14).

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i + B_i F_j) x(k) \tag{4.13}$$

$$x(k+1) = \sum_{i=1}^{r} h_i^2 G_{ii} x(k) + 2\sum_{i=1}^{r}\sum_{j=i+1}^{r} h_i h_j \left(\frac{G_{ij}+G_{ji}}{2}\right) x(k) \tag{4.14}$$

with,

$$G_{ij} = A_i + B_i F_j \tag{4.15}$$

Thus, following the procedure developed in Li *et al.* (2000):

$$\begin{aligned} \Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l x^T(k)[(G_{ij}+G_{ji})^T P(G_{kl}+G_{lk}) - 4P]x(k) \end{aligned} \tag{4.16}$$

Since, $A_j^T R A_i + A_i^T R A_j \leq A_i^T R A_i + A_j^T R A_j$, (4.16) can be written as (4.17).

$$\begin{aligned} \Delta V(x(k)) &\leq \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T(k)[(G_{ij}+G_{ji})^T P(G_{ij}+G_{ji}) - 4P]x(k) \\ &= \sum_{i=1}^{r} h_i^2 x^T(k)(G_{ii}^T P G_{ii} - P)x(k) + \\ &\quad 2\sum_{i=1}^{L}\sum_{j=i+1}^{L} h_i h_j x^T(k)\left[\frac{(G_{ij}+G_{ji})^T}{2} P \frac{(G_{ij}+G_{ji})^T}{2} - P\right] x(k) \end{aligned} \tag{4.17}$$

Hence, if (4.18) and (4.19) hold true:

$$G_{ii}^T P G_{ii} - P + W + F_i^T R F_i < 0 \tag{4.18}$$

$$\frac{(G_{ij}+G_{ji})}{2}P\frac{(G_{ij}+G_{ji})}{2}-P+W+\frac{F_i^TRF_j+F_j^TRF_i}{2} \tag{4.19}$$

then, (4.17) becomes,

$$\begin{aligned}\Delta V(x(k)) \leq & \sum_{i=1}^{r} h_i^2 x^T(k)(-W-F_i^TRF_i)x(k)+ \\ & 2\sum_{i=1}^{L}\sum_{j=i+1}^{L} h_ih_jx^T(k)\left[-W-\frac{F_i^TRF_j+F_j^TRF_i}{2}\right]x(k)\end{aligned} \tag{4.20}$$

Following Li *et al.* (2000), the term $x^T(k)Wx(k)+u^T(k)Ru(k)$ can be rewritten as (4.21).

$$x^T(k)\left[\sum_{i=1}^{r} h_i^2 x^T(k)(W+F_i^TRF_i)+2\sum_{i=1}^{r}\sum_{j=i+1}^{r} h_ih_j\left(W+\frac{F_i^TRF_j+F_j^TRF_i}{2}\right)\right]x(k) \tag{4.21}$$

Therefore,

$$\Delta V(x(k)) < -x^T(k)Wx(k)-u^T(k)Ru(k) \tag{4.22}$$

Which confirms that the Lyapunov function $V(x)$ is an upper bound of the cost function J_∞.

Moreover, the conversion to LMI format of the conditions given in (4.18) and (4.19) are made as follows:

Taking (4.18) and considering $P=\gamma Q^{-1}$:

$$(A_i+B_iF_i)^T\gamma Q^{-1}(A_i+B_iF_i)-\gamma Q^{-1}+W+F_i^TRF_i<0 \tag{4.23}$$

Defining $Y_i=F_iQ$, 4.23 can be described as .

$$(A_iQ+B_iY_i)^T\gamma Q^{-1}(A_iQ+B_iF_i)-\gamma Q+QWQ+Y_i^TRY_i<0 \tag{4.24}$$

Which is equivalent to (4.25), applying Schur procedure.

$$\begin{bmatrix} Q & * & * & * \\ A_iQ+B_iY_i & Q & * & * \\ W^{\frac{1}{2}}Q & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y_i & 0 & 0 & \gamma I \end{bmatrix} > 0 \tag{4.25}$$

Analogously, following Li *et al.* (2000), considering $P=\gamma Q^{-1}$ and $Y_i=F_iQ$ (4.19) is given by (4.26)

$$\begin{aligned}&4Q-(A_iQ+B_iY_j+A_jQ+B_jY_i)^TQ^{-1}(A_iQ+B_iY_j+A_jQ+B_jY_i)-4\gamma^{-1}QWQ- \\ &2\gamma^{-1}Y_i^TRY_i-2\gamma^{-1}Y_j^TRY_j>0\end{aligned} \tag{4.26}$$

Besides, applying the Schur complement (4.26) is given in (4.27).

$$\begin{bmatrix} 4Q & * & * & * & * \\ S & Q & * & * & * \\ 2W^{\frac{1}{2}}Q & 0 & \gamma I & * & * \\ \sqrt{2}R^{\frac{1}{2}}Y_i & 0 & 0 & \gamma I & * \\ \sqrt{2}R^{\frac{1}{2}}Y_j & 0 & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{4.27}$$

The LMIs previously proven assure the stability of the TS fuzzy system. In addition, input constraints can be added to the system. Hence, following Li *et al.* (2000) considering an input constraint represented by $\|u(k)\|_2 \leq u_{max}$,

$$\begin{aligned} \max_{k\geq 0}\|u(k)\|_2^2 &\leq \max_{z^T Q^{-1}, z<1}\left\|\sum_{i=1}^{r} h_i Y_i Q^{-1} z\right\|_2^2 \\ &\leq \max_{i}\max_{z^T Q^{-1}, z<1}\left\|\sum_{i=1}^{r} Y_i Q^{-1} z\right\|_2^2 \\ &\leq \lambda_{max}(Q^{-0.5} Y_i^T Y_i Q^{-0.5}) \end{aligned} \tag{4.28}$$

Thus, using Schur complement the imposed input constraint is:

$$\begin{bmatrix} Q & * \\ Y_i & u_{max}^2 I \end{bmatrix} > 0 \tag{4.29}$$

□

4.2 Offline fuzzy state observer

The next step in the controller development is the design of a state observer, this approach is often applied in systems where it is not possible to measure all the states of a model (PARK *et al.*, 2011). Consequently, Feng (2018) stated that in face of this impossibility it is necessary to design controls with output feedback, such as the observer-based method.

The state observer has the function of estimate the states of a system, thus circumventing the difficulty of measuring all states of a model. For the purpose of this study, an offline fuzzy state observer is utilized based on the developments of Feng (2018) and Tanaka e Wang (2001).

As the observer intends to minimize the errors between the estimated and the actual state variables, the proposition given in (4.30) must be ensured (TANAKA; WANG, 2001).

$$x(t) - \hat{x}(t) \rightarrow 0, \text{as } t \rightarrow \infty \tag{4.30}$$

with, $\hat{x}(t)$ as the estimated state vector.

Considering a system designed with a fuzzy controller associated with a fuzzy state observer and for a TS fuzzy model, the estimated system is represented in (4.31) (TANAKA; WANG, 2001).

$$\begin{aligned}\hat{x}(k+1) &= \sum_{j=1}^{r} h_j(z(k))(A_j\hat{x}(k)+B_ju(k)+L_j(\hat{y}(t)-y(t))\\ \hat{y}(k) &= \sum_{j=1}^{r} h_j(z(k))(C_j\hat{x}(k)+D_ju(k)\end{aligned} \tag{4.31}$$

where, the weight $h_j(z(k))$ is the one obtained by the TS fuzzy model presented in (2.12), and L_j is the observer gain for each fuzzy rule.

Moreover, for an observer-based design the PDC control law given in (2.17) becomes:

$$u(k) = -\left(\sum_{j=1}^{r} h_j(z(k))F_j\right)\hat{x}(k) \tag{4.32}$$

Therefore, the fuzzy observer error can be expressed as follows,

$$e(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))\left(A_i - F_iC_j\right)e(k) \tag{4.33}$$

Hence, following the offline state observer proposed by Feng (2018) Theorem 2 is defined.

Theorem 2 (Offline fuzzy state observer (FENG, 2018))**.** *The fuzzy observer gains L_j are obtained if there is a positive define matrix P, and a set of matrices R_i, that satisfy the inequality given in* (4.34)*:*

$$\begin{bmatrix} -P & A_i^TP + C_j^TR_i^T \\ PA_i + R_iC_j & -P \end{bmatrix} < 0, \tag{4.34}$$

In this way, if this optimization problem is solved, the gain of the observer that guarantees the stabilization of $\hat{x}(k)$ is given by (4.35).

$$L_j = P^{-1}R_j \tag{4.35}$$

Proof. Taking $G_{ij} = A_i + F_iC_j$, the global stability of the error (4.33) is assured if the following LMI is satisfied.

$$G_{ij}PG_{ij} - P < 0 \tag{4.36}$$

Using the Schur complement and replacing $G_{ij} = A_i + F_i C_j$ is express as (4.37).

$$\begin{bmatrix} -P & A_i^T P + C_j^T F_i^T P \\ PA_i + PF_i C_j & -P \end{bmatrix} < 0, \tag{4.37}$$

Defining $R_i = PF_i$,

$$\begin{bmatrix} -P & A_i^T P + C_j^T R_i^T \\ PA_i + R_i C_j & -P \end{bmatrix} < 0, \tag{4.38}$$

□

4.3 Stability criteria

The observer-based output feedback is characterized by a separated design of a state feedback controller and an estimator. Furthermore, following Kim *et al.* (2006) this approach often leads to difficulties in guaranteeing the overall system stability. In view of this problem, Wan e Kothare (2002) proposed a methodology to ensure the robust stability of the association controller-observer, which consists of a criteria that evaluates the closed-loop system feasibility.

Inspired by the procedure developed by Wan e Kothare (2002), this dissertation proposes a new stability criteria to guarantee the closed-loop stability of the proposed observer-based output feedback FMPC. In contrast with the work developed by Wan e Kothare (2002), this study considers a T-S fuzzy model, a fuzzy MPC controller and an offline fuzzy state observer. In addition, the stability criteria are designed considering an online and offline approach of the controller, which are described in subsections 4.3.1 and 4.3.2, respectively.

4.3.1 Stability criteria design for online approach

Considering an online procedure of the FMPC controller, the fuzzy gains $F_j(k)$ (4.9) and state matrices $A(k), B(k), C(k)$ and $D(k)$ are determined for each iteration. Moreover, the observer fuzzy gains are obtained offline and are given by L_j (4.35). Thus, the closed-loop augmented system for the controller-observer union is given by (4.39).

$$\mathscr{X}(k+1) = \mathscr{A}_{poly}(k)\mathscr{X}(k) \tag{4.39}$$

where, $\mathscr{X} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$ and $\mathscr{A}_{poly}(k) = \begin{bmatrix} A(k) & B(k)F_j(k) \\ L_j C(k) & A(k) + B(k) - L_j C(k) \end{bmatrix}$

The structured defined in (4.39) aims to assure the closed-loop stability criteria stated in Theorem 3.

Theorem 3 (Robust stability criteria for online observer-based output feedback for T-S Fuzzy MPC)**.** *The augmented system* (4.39) *is robustly stable if exists the matrix* $\mathscr{Q} > 0$*, with compatible dimension, such that for all state matrices and fuzzy gains the LMI given in* (4.40) *holds true.*

$$\begin{bmatrix} \mathscr{Q} & \mathscr{Q}\mathscr{A}_{poly}(k)^T \\ \mathscr{Q}\mathscr{A}_{poly}(k) & \mathscr{Q} \end{bmatrix} > 0 \tag{4.40}$$

Proof. If (4.40) is satisfied, at each sample time, for all state matrices and fuzzy gains, then taking $\mathscr{P} = \mathscr{Q}^{-1}$ and using Schur complement method:

$$\mathscr{P} - \mathscr{A}_{poly}(k)^T \mathscr{P}\mathscr{A}_{poly}(k) > 0 \tag{4.41}$$

Therefore, assuring that the Lyapunov quadratic function $\mathscr{X}^T \mathscr{P} \mathscr{X}$ is monotonically decreasing, i.e., the system is asymptotically stable. □

4.3.2 Stability criteria design for offline approach

For the offline method, the FMPC controller presents a fixed set of fuzzy gains F_j as well as the observer gains L_j and state matrices $A(k)$,$B(k)$,$C(k)$ and $D(k)$ are determined for each sample time. In that sense, the closed-loop augmented system considering the controller-observer interaction is given by (4.42).

$$\mathscr{X}(k+1) = \mathscr{A}_{poly}(k)\mathscr{X}(k) \tag{4.42}$$

with,

$$\mathscr{X} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \tag{4.43}$$

and,

$$\mathscr{A}_{poly}(k) = \begin{bmatrix} A(k) & B(k)F_j \\ L_jC(k) & A(k)+B(k)-L_jC(k) \end{bmatrix} \tag{4.44}$$

Thus, Theorem 4 explains the closed-loop stability for the offline approach.

Theorem 4 (Robust stability criteria for offline observer-based output feedback for T-S Fuzzy MPC)**.** *The augmented system* (4.42) *is robustly stable if exists the matrix* $\mathscr{Q} > 0$*, with compatible dimension, such that for all state matrices, controller fuzzy gains set and observer fuzzy* *gains,*

$$\begin{bmatrix} \mathscr{Q} & \mathscr{Q}\mathscr{A}_{poly,i,j}^{T} \\ \mathscr{Q}\mathscr{A}_{poly,i,j} & \mathscr{Q} \end{bmatrix} > 0 \tag{4.45}$$

where, $\mathscr{A}_{poly,i,j} = \begin{bmatrix} A_i & B_iF_j \\ L_j & A_i + B_i - L_jC_i \end{bmatrix}$, $i = 1,...,N$ *and* $j = 1,...,r$.

Proof. If (4.45) is satisfied for all the polytopic vertices from the set Ω of the system and all fuzzy gains from the sets F_j and L_j, then for an arbitrary set $[A(k)\ B(k)] \in \Omega$ with the controller and observer fuzzy gains $F_j(k)$ e $L_j(k)$,

$$\begin{bmatrix} \mathscr{Q} & \mathscr{Q}\mathscr{A}_{poly}(k)^{T} \\ \mathscr{Q}\mathscr{A}_{poly}(k) & \mathscr{Q} \end{bmatrix} > 0 \tag{4.46}$$

Hence, defining $\mathscr{P} = \mathscr{Q}^{-1}$ and using Schur complement, it is possible to affirm that,

$$\mathscr{P} - \mathscr{A}_{poly}(k)^{T}\mathscr{P}\mathscr{A}_{poly}(k) > 0 \tag{4.47}$$

Thus, as for Theorem 3, the quadratic Lyapunov function $\mathscr{X}^{T}\mathscr{P}\mathscr{X}$ is invariably decreasing, i.e., the system is asymptotically stable. □

Defining the stability criteria for the controller-observer union is the last step of the proposed controller methodology. Thus, Sections 4.4 and 4.5 resumes the general proposed procedure for the online and offline approaches, respectively.

4.4 Observer-based output feedback FMPC methodology

For the online procedure, the controller optimization problem defined in (4.4) is solved at each sample time, resulting in a fuzzy gains set $F_i(k)$. However, the observer gains are predetermined since its optimization procedure is made offline. Moreover, the stability criteria is implemented as the FMPC, i.e., at each sample time the criteria is applied. Thus, considering the above, the Theorem 5 resumes the online procedure for the proposed control system.

Theorem 5 (Online Observer-based Output Feedback TS FMPC with guaranteed closed-loop stability)**.** *For a T-S fuzzy model* (2.10)*, considering the PDC control law* (2.17) *with the state feedback fuzzy gains given in* (4.9)*, and the state observer fuzzy gains* (4.35)*, the observer-based* *output feedback T-S FMPC is asymptotically stable if the minimization problem expressed in* (4.48) *is solved at each sample time.*

$$\min_{\gamma, Y_j, Q} \gamma, \tag{4.48}$$

subject to (4.5)-(4.34), (4.39) *and* (4.40).

4.5 Offline Observer-based output feedback FMPC methodology

The offline methodology is proposed as a way to reduce the computational effort that makes the online implementation more time consuming. For this purpose, unlike the online procedure, the offline methodology applies the FMPC controller and the stability criteria procedure for a set of points, which results in a predefined set of fuzzy controller gains F_j, as well as the observer fuzzy gains L_j. A fixed gain is chosen among the fuzzy gains set and then is applied to the iterated procedure.

The development of an offline approach for MPC controllers using LMIs was made by Wan e Kothare (2003), their work is based on the the asymptotically stable invariant ellipsoid concept, which are explained and detailed in Subsection 4.5.1.

4.5.1 Stability Ellipsoids

The invariant ellipsoid concept is used in offline MPC methods in order to guarantee the system stability. Following Wan e Kothare (2002), this is achieved by limiting the system states forming asymptotically stable invariant ellipsoids. Souza (2015) states that the system states became limited by an ellipsoid when submitted to the constraint given (4.5). Furthermore, the system stability is assured through the asymptotically stable invariant ellipsoid defined as follows.

Definition 1 (Asymptotically stable invariant ellipsoid (WAN; KOTHARE, 2003))**.** *Considering a discrete system* $x(k+1) = f(x(k))$*, a subset* $\mathscr{E} = \left\{x \in \mathscr{R}^{n_x} | x^T Q^{-1} x \leq 1\right\}$ *of the state space* $\mathscr{R}^{n_x}$ *is an asymptotically stable invariant ellipsoid, if for* $x(k_1) \in \mathscr{E}$*,* $x(k) \in \mathscr{E}$ *and* $x(k) \to 0$ *as* $k \to \infty$*, with* $k \geq k_i$*.*

Moreover, considering a model as (2.10) *with a control law as* (2.17) *which solves the optimization problem given in* (4.4) *for a state* x_0*. Then the subset* $\mathscr{E} = \left\{x \in \mathscr{R}^{n_x} | x^T Q^{-1} x \leq 1\right\}$ *of the state space* $\mathscr{R}^{n_x}$ *is an asymptotically stable invariant.*

Proof. From Wan e Kothare (2003) it can be seen that: Since the only state dependent LMI from (4.4) is (4.5), which is satisfied for all states inside de ellipsoid. Then, the minimization considering the state x_0 results in feasible matrices γ, Y_j, Q for any other state in $\mathscr{E}$.

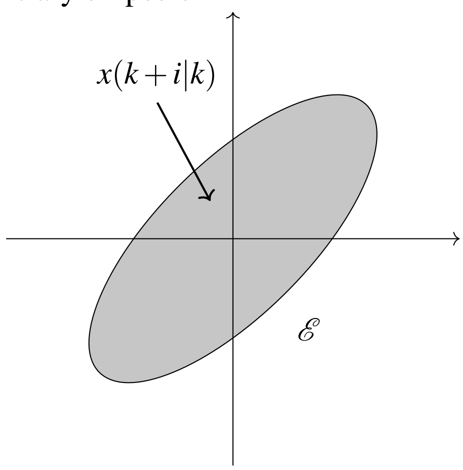
Therefore, applying the control law $u(k) = -\left(\sum_{j=1}^{r} h_j(z(k))Y_jQ^{-1}\right)x(k)$ to a state $\tilde{x}(k) \in \mathscr{E} \neq x_0 \neq 0$, which satisfy (4.6)-(4.8),

$$\tilde{x}(k+i+1)^T Q^{-1}\tilde{x}(k+i+1) < \tilde{x}(k+i)^T Q^{-1}\tilde{x}(k+i), \;\; i \geq 0 \tag{4.49}$$

Hence, proving that $\tilde{x}(k+i) \in \mathscr{E}$ and $\tilde{x}(k+i) \to 0$ as $k \to \infty$. □

The invariant ellipsoids can be interpreted as a geometric bound for the robust system stability, as each matrix Q^{-1} can be designed as an ellipse or ellipsoid (according to its dimensions). A 2-dimensions representation of an arbitrary ellipsoid is illustrated in Figure 10.

Figure 10 – 2-dimensions arbitrary ellipsoid



Source: Adapted from Kothare *et al.* (1996)

As stated by Costa (2017), physically the invariant ellipsoid stability can read as: a BIBO stable response stays within the ellipsoid boundaries and this response tends to zero on steady state, considering the Finite Impulse Response (FIR), for a given initial conditions. Moreover if the states are limited by an ellipsoid, the optimization problem (4.4) establishes a control law which assures that the future states are also limited by an ellipsoid, with a smaller ratio.

Thus, considering the concepts introduced above, the offline procedure for the proposed control system is resumed in Theorem 6.

Theorem 6 (Offline Observer-based Output Feedback TS FMPC with guaranteed closed-loop stability)**.** *For an offline system, given an initial feasible condition x_2, a sequence of minimizers $(\gamma, Q, Y_i, Y_j, \mathscr{Q})$ is calculated following* (4.4)-(4.9)*,* (4.34) *and* (4.45)*. Take k:=1*

1. *Compute the minimizers* $(\gamma_k, Q_k, Y_{ik}, Y_{jk}, \mathscr{Q}_k)$ *with the additional constraints* $Q_{k-1} > Q_k$ *and and keep* $Q_k^{-1}, Y_{ik}, Y_{jk}, F_{ik}$ *and* F_{jk} *in a lookup table;*
2. *If* $k < N$*, choose the state* x_{k+1} *satisfying* $\|x_{k+1}\|_{Q^{-1}} \leq 1$*. Take k:=k+1 and go to step one.* ***Lookup table****: given the initial condition* $\|x(0)\|_{Q^{-1}} \leq 1$ *take the state* $x(k)$ *for the respective time k. Plot the search around* Q^{-1} *in the lookup table to find the biggest k (or the smallest ellipsoid).*
3. *Apply the control law* (4.32)*.*

4.6 Chapter's Summary

This chapter described the core of this dissertation, defining the main methodologies used to implement the proposed output feedback FMPC approach. Initially, the observer-based output feedback method from Wan e Kothare (2002) was introduced as a three step mechanism, first a novel state-feedback controller was proved based on the work developed by Li *et al.* (2000). Next, the fuzzy state observer proposed by Feng (2018) was discussed, and finally new stability criteria for the controller-observer augmented system was proposed. Furthermore, two overall procedures was presented in the form of theorems for an online and an offline approach of the proposed control method.

5 NUMERICAL EXAMPLE

This chapter presents an application of the proposed control strategy described in the above chapters. This application involves the benchmark model proposed by Park *et al.* (2011) designed through the TS fuzzy methodology (2.10)-(2.15) and adding the proposed observer-based output feedback FMPC, considering the online and offline approaches according to Theorems 5 and 6.

Hence, this chapter is organized as follows: first, Section 5.1 describes the process plant and develops its TS fuzzy model, then Section 5.2 introduces the control configuration including the block diagram for the proposed method. Moreover, Section 5.3 resumes the main results for the online and offline applications, and finally Section 5.4 discusses the contributions of the chapter.

5.1 Model description

The plant used is the numerical example described in the work of Park *et al.* (2011) as a Linear Parameter Varying (LPV) system, with its state-space equations described in (5.1).

$$\begin{aligned} x(k+1) &= A(\alpha(k))x(k) + B(\beta(k))u(k), \\ y(k) &= Cx(k), \end{aligned} \tag{5.1}$$

where $A(\alpha(k))$, $B(\beta(k))$ and C are the matrices (5.2), (5.3) and (5.4), respectively.

$$A(\alpha(k)) = \begin{bmatrix} 0.872 & -0.0623\alpha(k) \\ 0.0935 & 0.997 \end{bmatrix} \tag{5.2}$$

$$B(\beta(k)) = \beta(k) \begin{bmatrix} 0.0935 \\ 0.00478 \end{bmatrix} \tag{5.3}$$

$$C = \begin{bmatrix} 0.333 & -1 \end{bmatrix} \tag{5.4}$$

The parameters α and β are in the ranges:

$$\alpha(k) \in [1,\ 5] \quad \text{and} \quad \beta(k) \in [0.1,\ 1]. \tag{5.5}$$

In order to implement the proposed control approach, this LPV system is modeled using the TS fuzzy approach, as in (2.10). Thus, considering that the state matrices are functions of the parameters α and β, two random values for each of these parameters are defined at each sampling time, being α_1, α_2, β_1 and β_2. Therefore, following the limits given in (5.5), the parameters α_1, α_2, β_1 and β_2 vary inside the regions:

$$\begin{aligned} &\alpha_1(k) \in [1,\ 2.5], \qquad \beta_1(k) \in [0.1,\ 0.55]. \\ &\alpha_2(k) \in [2.5,\ 5], \qquad \beta_2(k) \in [0.55,\ 1]. \end{aligned} \tag{5.6}$$

Hence, the state-space model given in (5.1) can be written as a TS fuzzy model with two if-then rules, as expressed by (5.7) and (5.8).

$$\begin{aligned} &Rule\ 1 : IF\ \ x_1(k) = \mu_1 \\ &THEN : \begin{cases} x(k+1) = A(\alpha_1(k))x(k) + B(\beta_1(k))u(k), \\ y(k) = Cx(k). \end{cases} \end{aligned} \tag{5.7}$$

$$\begin{aligned} &Rule\ 2 : IF\ \ x_1(k) = \mu_2 \\ &THEN : \begin{cases} x(k+1) = A(\alpha_2(k))x(k) + B(\beta_2(k))u(k), \\ y(k) = Cx(k). \end{cases} \end{aligned} \tag{5.8}$$

Note that the system states are used as premise variables for the design of the membership functions, which are chosen as the nonlinear membership functions μ_1 and μ_2, given in (5.9), based on the study developed by Xia *et al.* (2010).

$$\begin{aligned} &\mu_1(x_2(k)) = \frac{1 + sin(x_2)}{2}, \\ &\mu_2(x_2(k)) = \frac{1}{1 + e^{x_2}}. \end{aligned} \tag{5.9}$$

5.2 Controller design

The proposed controller procedure for the numerical example is resumed in the block diagram illustrated by Figure 11. The system states are given by:

$$x(k+1) = A_z(k)x(k) + B_z(k)u(k) \tag{5.10}$$

Also, the estimated states are given as,

$$\hat{x}(k+1) = A_z(k)\hat{x}(k) + B_z(k)u(k) + L_z(\hat{y}(k) - (y(k))) \tag{5.11}$$

with, $A_z(k)$ and $B_z(k)$ obtained by the TS fuzzy design of $A(\alpha_i(k))$ and $B(\beta_i(k))$, and L_z follows the same procedure for the gains L_j.

Besides, using PDC principle, the control law is given in 5.12,

$$u(k) = -F_z(k)\hat{x}(k) \tag{5.12}$$

where $F_z(k)$ is obtained by the TS fuzzy association of the gains F_j.

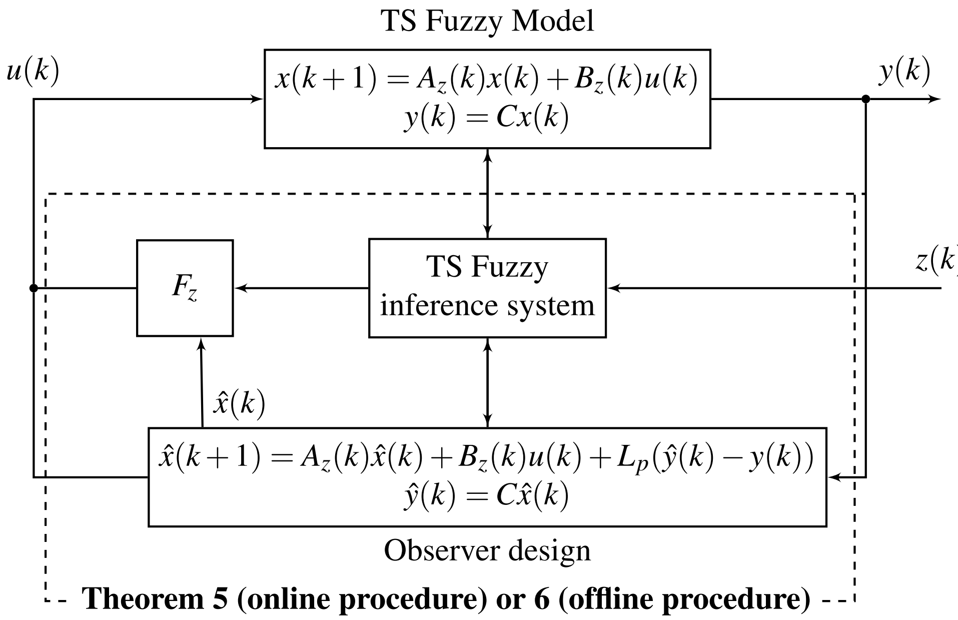
Considering that the output and the estimated output are,

$$\begin{aligned} y(k) &= Cx(k) \\ \hat{y}(k) &= C\hat{x}(k) \end{aligned} \tag{5.13}$$

the controller-observer state-equations for this application are resumed in (5.14).

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} A_z(k) & -B_z(k)F_z(k) \\ -L_zC & A_z(k) - B_z(k)F_z(k) + L_zC \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix} \tag{5.14}$$

Figure 11 – Block Diagram for the numerical example



Source: The Author (2020)

Furthermore, the controller is defined using the weighting matrices from (4.6)-(4.7), which are designed as (5.15).

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = 1 \tag{5.15}$$

Also, an input constraint is added as $|u(k)| < 1,\ k \geq 0$, for the initial states $x = [-1.5\ \ -0.2]^T$ and $\hat{x} = [-0.5\ \ 1]^T$. Considering these parameters and the process model, the obtained fuzzy state observer gains are given in (5.16).

$$
\begin{aligned}
L_1 &= [-0.1831\quad 0.9231]^T \\
L_2 &= [-0.4156\ -0.9210]^T
\end{aligned}
\tag{5.16}
$$

5.3 Simulation results

This section discusses the results obtained from the computational simulation of the proposed output feedback FMPC applied to the model given in Section 5.1. This simulation adopts the toolboxes YALMIP and the SEDUMI solver to implement the LMIs. Moreover, the analysis is divided into online and offline approaches, which are presented in subsections 5.3.1 and 5.3.2.

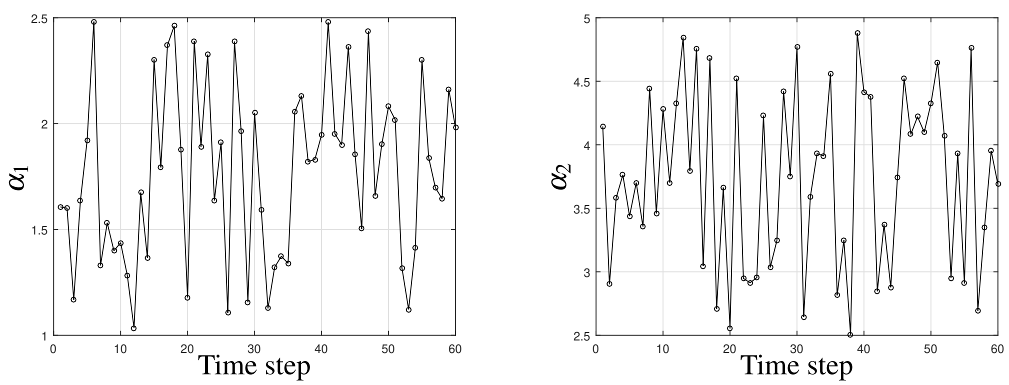
5.3.1 Online approach for numerical example

The performance analysis of the proposed output feedback FMPC is made in comparison with the output feedback MPC from Kim e Lee (2017) and Rego (2019), the latter being an improvement on the former. The responses over time, poles allocation and performance indexes are used to evaluate this comparison, which are illustrated and discussed as follows.

Considering the exposed in Section 5.1, the random variation for the parameters α and β are illustrated in Figures 12 and 13.

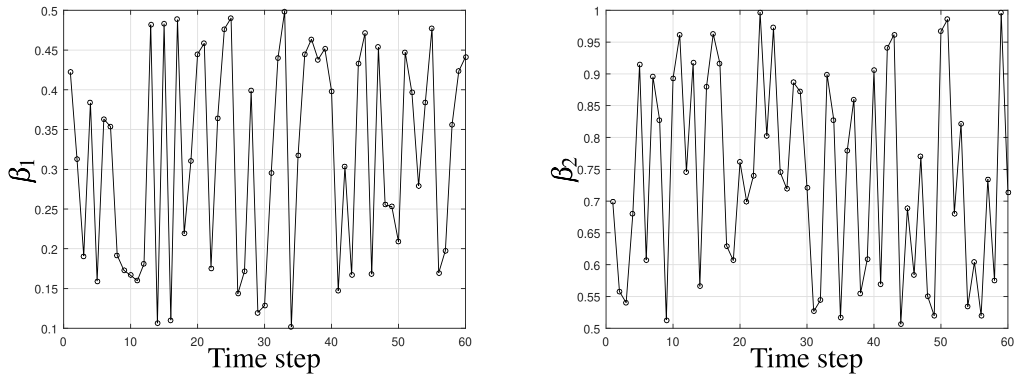
Figure 12 – Random variation for the parameter α

(a) α_1 (b) α_2



Source: The Author (2020)

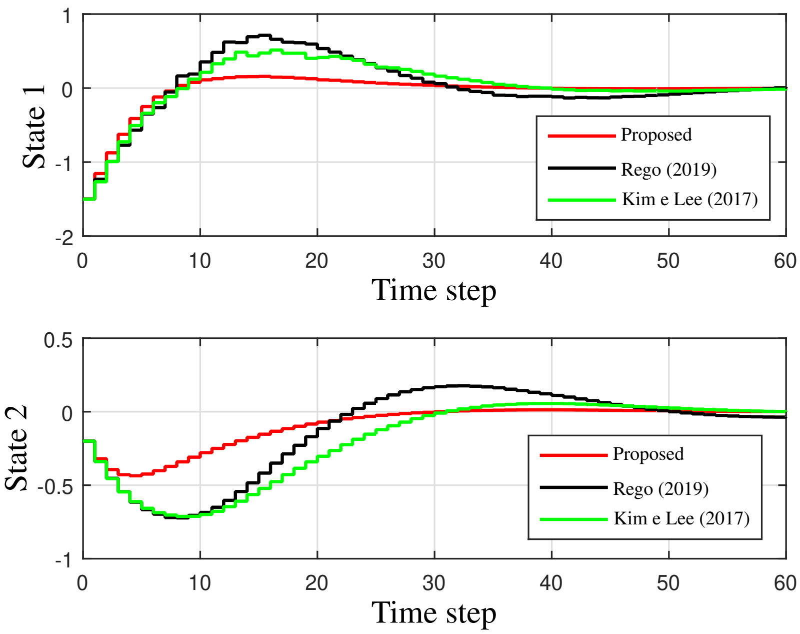
Figure 13 – Random variation for the parameter β
(a) β_1 (b) β_2



Source: The Author (2020)

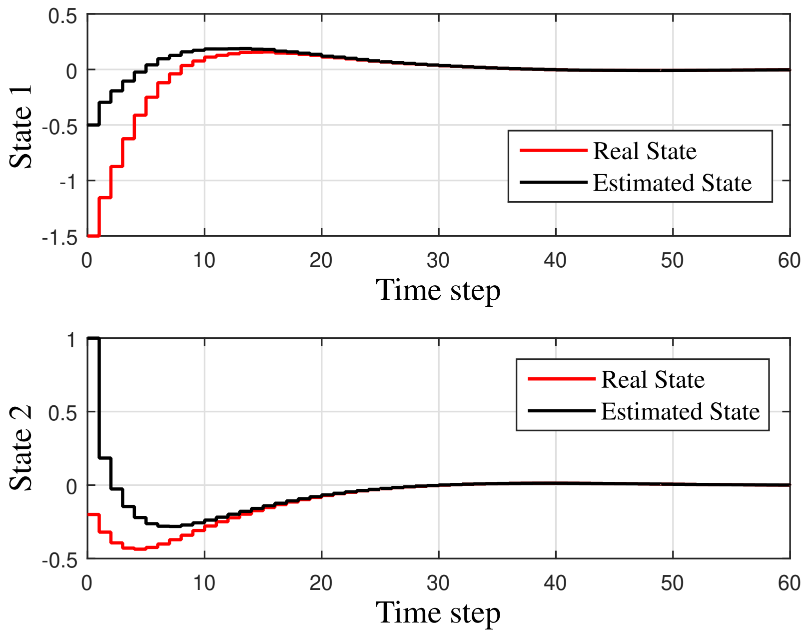
In view of this, Figure 14 displays the system states time response, considering the comparison between the proposed controller and the controllers from Rego (2019) and Kim e Lee (2017). From this figure, it can be noted that the controllers are able to stabilize the system over time. However, the proposed output feedback FMPC presents less oscillations, a lower overshoot and a a faster settling time in contrast with Rego (2019) and Kim e Lee (2017). Moreover, Figure 15 prints the time response for the real and estimated states considering the proposed output feedback FMPC approach. It is possible to highlight the proper functioning of the fuzzy state observer, since the estimated states $\hat{x}(k)$ converge to real system states $x(k)$ over time.

Figure 14 – System states time response.



Source: The Author (2020)

Figure 15 – Real × Estimated states for the proposed controller



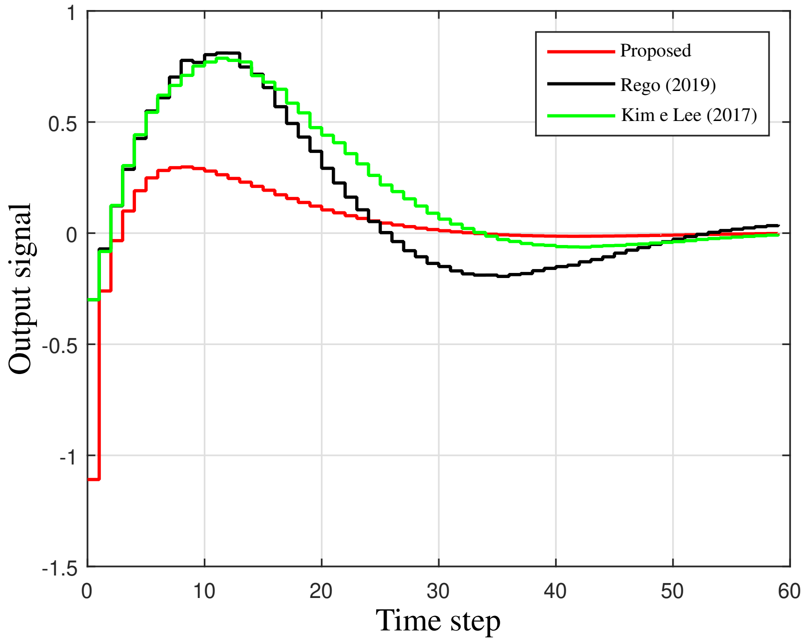
Source: The Author (2020)

Besides, the output responses for the control systems are represented in Figure 16. Note that, as with system states, the output response is more stable and stabilize faster for the proposed controller. The overshoot from the curves of Rego (2019) and Kim e Lee (2017) are more than twice the proposed controller overshoot, and the proposed procedure does not cause undershoot, unlike Rego (2019) and Kim e Lee (2017). Which confirms the superiority of the proposed output feedback FMPC.

The control efforts to reach those output curves are displayed in Figure 17. From this figure, it can be seen that all studied techniques satisfy the imposed input constraint. In addition, the control signal for the proposed controller has a faster and less oscillating stabilization than the one from Kim e Lee (2017). However, the control signal from Rego (2019) presents a better performance. Considering the exposed control effort, the time response for the objective function $\gamma(k)$, given in (4.4) is exhibited in Figure 18, showing a comparison between the studied controllers. It is possible to perceive through this figure that the proposed controller and the one from Rego (2019) acts similarly in therms of $\gamma(k)$. Nevertheless, the output feedback MPC from Kim e Lee (2017) presents a slower stabilization, and a much higher maximum value for this variable.

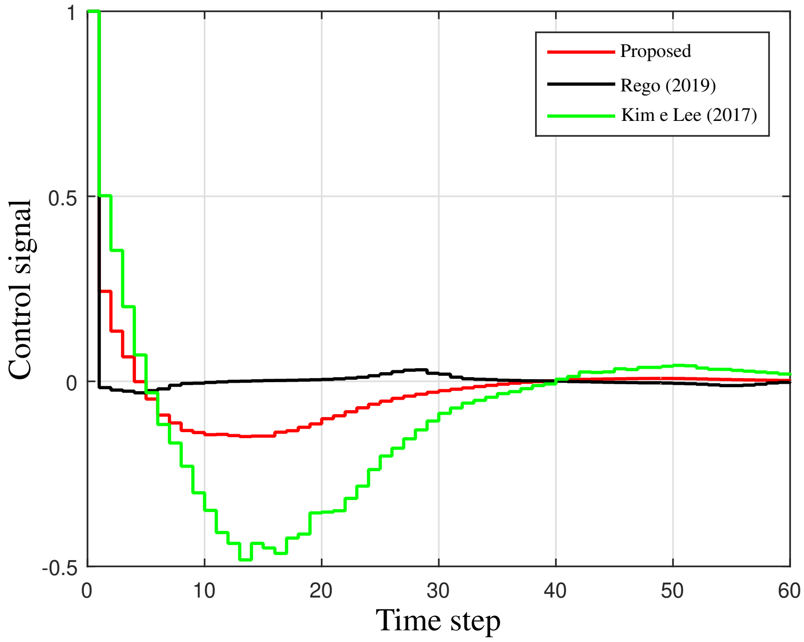
Furthermore, the presented control laws are analyzed through the allocation of the system poles in the unit circle. This analysis makes it possible to conclude whether or not the

Figure 16 – Output signal y(k).



Source: The Author (2020)

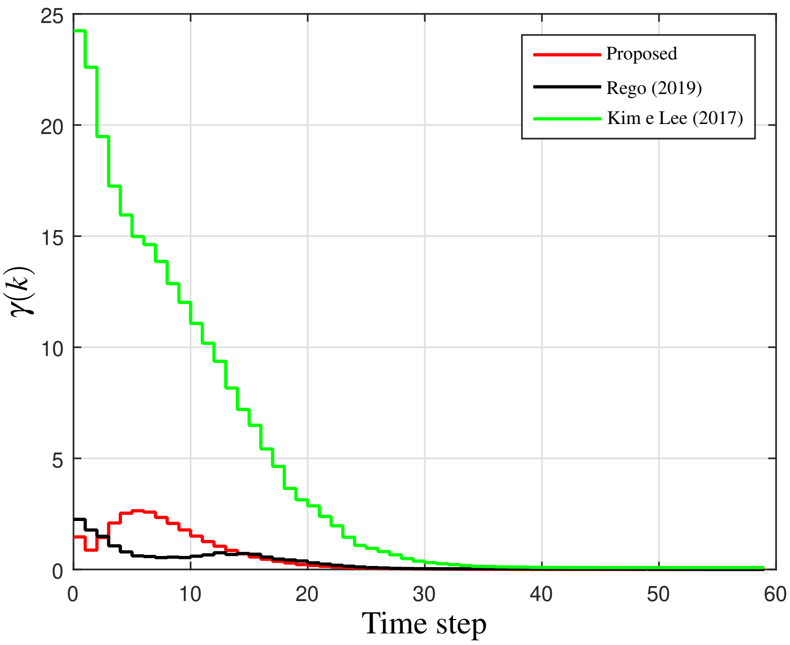
Figure 17 – Control signal u(k).



Source: The Author (2020)

controlled system is stable, considering if all poles are within the unit circle. The obtained poles from the controllers are shown in Figure 19, where Figure 19a illustrates the complete unit circle and Figure 19b displays an approximated view. As it can be seen, for all studied controllers the poles are placed within the unit circle, i.e. the controllers present a stable response.

Figure 18 – Objective function.

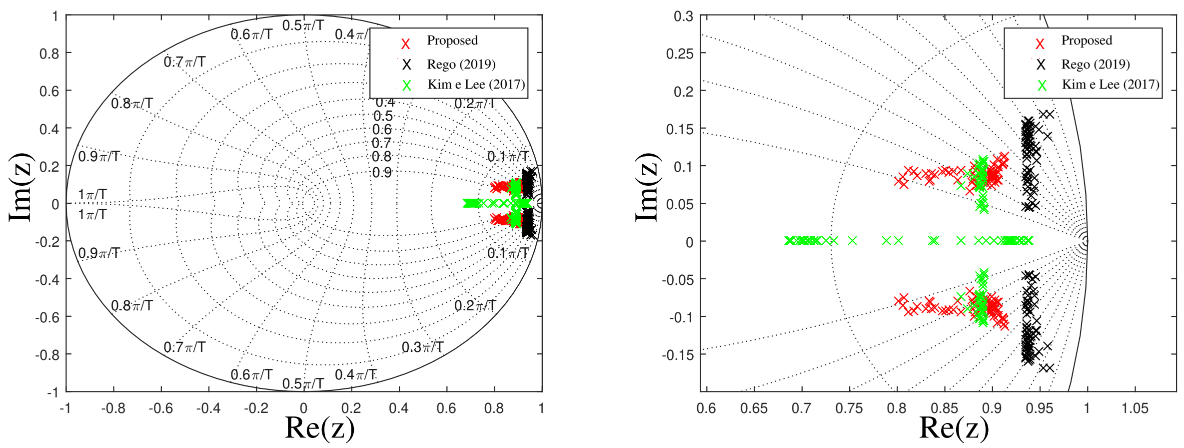


Source: The Author (2020)

Nonetheless, some poles from Rego (2019) and Kim e Lee (2017) controllers are closer to the unit circle extreme than the poles from the proposed output feedback FMPC method.

Figure 19 – Poles allocation in the z-plane

(a) Poles allocation (b) Approximated view



Source: The Author (2020)

Considering all the time responses for the parameters previously analyzed, Table 1 resumed the variation intervals for the proposed procedure in comparison with Rego (2019) and Kim e Lee (2017), showing superior results by the methodology proposed in this dissertation

Table 1 – Variation intervals for the studied parameters

	Proposed	Rego (2019)	Kim e Lee (2017)
x_1	$[-1.5,\ 0.158]$	$[-1.5,\ 0.7129]$	$[-1.5,\ 0.5144]$
x_2	$[-0.4361,\ 0.01271]$	$[-0.7226,\ 0.1762]$	$[-0.7124,\ 0.05577]$
y	$[-1.109,\ 0.2979]$	$[-0.2995,\ 0.8106]$	$[-0.2995,\ 0.7871]$
u	$[-0.1492,\ 1]$	$[-0.03096,\ 1]$	$[-0.4824,\ 1]$
γ	$[0,\ 2.641]$	$[0,\ 2.254]$	$[0,\ 24.24]$
Poles Re(z)	$[0.8011,\ 0.9136]$	$[0.9346,\ 0.9608]$	$[0.6863,\ 0.9385]$
Poles Im(z)	$[-0.1126,\ 0.1126]$	$[-0.1679,\ 0.1679]$	$[-0.1083,\ 0.1083]$

Source: The Author (2020)

Ultimately, the performance of the studied controllers are evaluated using some performance indices, which are Integrated Absolute Error (IAE), Integral of Squared Error (ISE), Integral of Time-weighted Absolute Error (ITAE), Integral of Time-weighted Squared Error (ITSE) and the cost function J_∞, given in (5.17)-(5.20) and (4.2), respectively. The results are resumed in Table 2, in which is possible to affirm that the proposed controller has a better performance in terms of all evaluated metrics.

$$IAE = \sum_{i=1}^{N_k} (ref_i - y_i) \tag{5.17}$$

$$ISE = \sum_{i=1}^{N_k} (ref_i - y_i)^2 \tag{5.18}$$

$$ITAE = \sum_{i=1}^{N_k} i(ref_i - y_i) \tag{5.19}$$

$$ITSE = \sum_{i=1}^{N_k} i(ref_i - y_i)^2 \tag{5.20}$$

where, N_k is the last simulation point, ref_i represents the desired reference and y_i is the output response.

Table 2 – Performance indices for the studied controllers

	Proposed	Rego (2019)	Kim e Lee (2017)
IAE	5.9098	15.1623	14.9418
ISE	2.2159	7.7246	8.0794
ITAE	62.2817	258.3642	227.9877
ITSE	9.6937	96.5466	100.4144
J_∞	8.3458	18.1292	19.6421

Source: The Author (2020)

5.3.2 Offline approach for numerical example

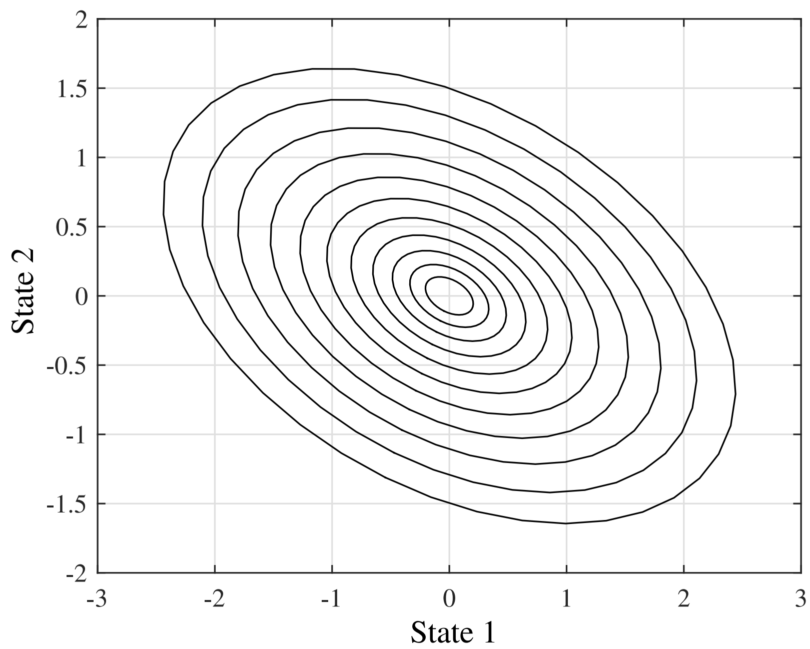
The offline approach for the output feedback FMPC are obtained using the stability invariant ellipsoid concept, as discussed in Section 4.5. Therefore, the results analysis is made in term of this ellipsoids, the obtained time response, closed-loop poles stability in z-map and performances indexes, in comparison with the proposed online procedure.

Thus, following the steps from Theorem 6, the stability invariant ellipsoids are geometric representation of the matrices Q, which are obtained from (4.5)-(4.8) and store in a lookup table, considering a set of ten points x_{set} obtained from the system states. Besides the matrices Q, the lookup table also stores the fuzzy gains for the proposed controller.

Therefore, applying the offline proposed procedure for the TS model described in Section 5.1, the set of matrices Q_k, with $k = 1, 2, ..., 10$ are obtained. Which are illustrated in Figure 20, and their respective gains F_{jk} and points $x_{set}(k)$ are listed in Table 3. Furthermore, Figure 21 presents the geometric projection of those ellipsoids over time and in contrast with x_{set}. The analysis of Figures 20 and 21 shows that the size of the ellipsoids decreases as i reaches 20, thus inferring the tendency to stabilize the ellipsoids.

According to Costa (2017), the stability of the system for the offline stability invariant ellipsoids approach is also confirmed if the impulse response for the nominal operating point remains within the limits of the ellipsoid, and if that response tends to zero in a steady state. Moreover, considering the already proven stability tendency of the ellipsoid Q_k, the closed loop system is stable for any value of k. Thus, the choice of the implemented gain is up to the controller designer. Choosing $k = 10$, the geometric projection of Q_{10} and the impulse response are illustrated in Figure 22. It is possible to notice that the impulse response is restricted inside the limits of the ellipsoid. In addition, the impulse response converge to the origin, thus it can be concluded that the proposed controller guarantees the system stability.

Figure 20 – Stability invariant ellipsoids Q_k.



Source: The Author (2020)

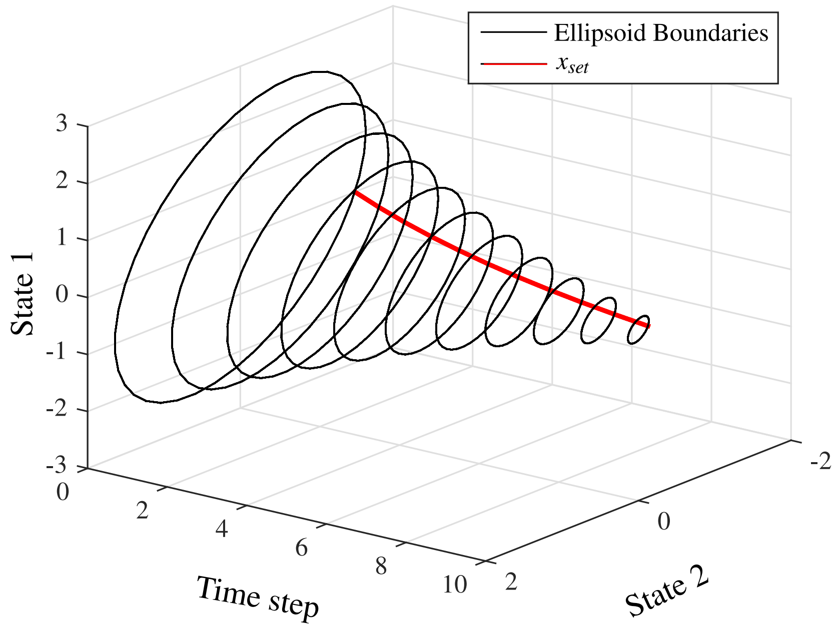
Table 3 – Set of points and fuzzy gains for the offline procedure

k	$\mathbf{x_{set}}$	F_1	F_2
1	-1.5	[-0.0436 -0.0066]	[-0.1441 0.0813]
2	-1.2955	[-0.0535 0.0011]	[-0.1530 0.0677]
3	-1.1086	[-0.0621 0.0076]	[-0.1591 0.0583]
4	-0.9380	[-0.0703 0.0140]	[-0.1667 0.0467]
5	-0.7829	[-0.0740 0.0168]	[-0.1747 0.0344]
6	-0.6423	[-0.0795 0.0210]	[-0.1836 0.0207]
7	-0.5153	[-0.0825 0.0234]	[-0.1898 0.0113]
8	-0.4009	[-0.0845 0.0249]	[-0.1940 0.0048]
9	-0.2983	[-0.0854 0.0255]	[-0.1972 -0.0001]
10	-0.2067	[-0.0841 0.0245]	[-0.2000 -0.0044]

Source: The Author (2020)

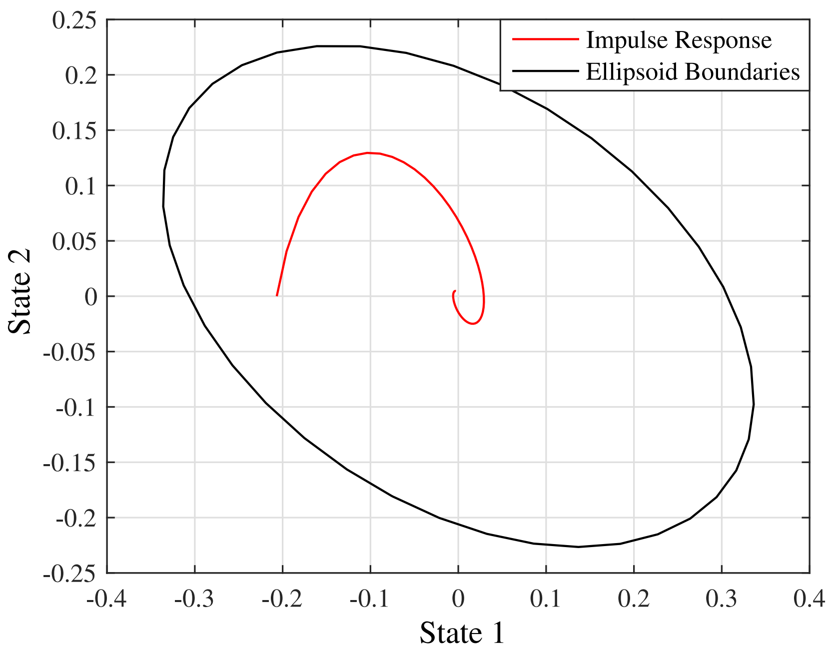
Furthermore, applying the chosen gain for $k = 10$, the system time responses are then obtained. Which are illustrated in Figures 23-26 in comparison with the proposed online output feedback FMPC approach. From these figures, it is possible to affirm the ability of the offline method to stabilize the system over time, with a performance very similar to that of the online approach, for all evaluated parameters. The offline procedure was also able to conduct the imposed input constraint and to perform stable considering the z-plane poles allocation. The similarity of both methods are also validated by the performance indices presented in Section 5.3.1, which are given in Table 4.

Figure 21 – Ellipsoids over time × x_{set}



Source: The Author (2020)

Figure 22 – Invariant ellipsoid × impulse response.



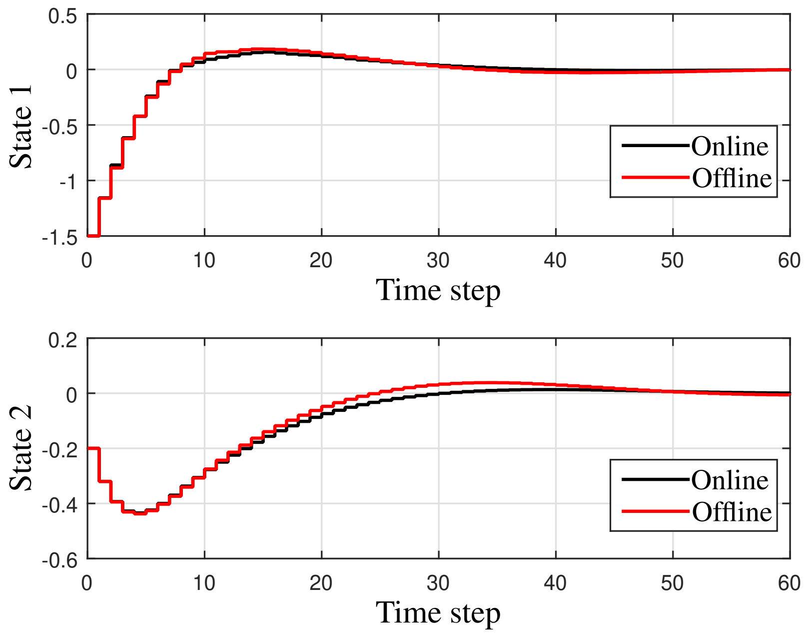
Source: The Author (2020)

Table 4 – Online × Offline Performance indices

	Online	Offline
IAE	5.9416	6.1558
ISE	2.2091	2.2192
ITAE	63.5866	74.2704
ITSE	9.7335	10.0279
J_∞	8.2727	8.1271

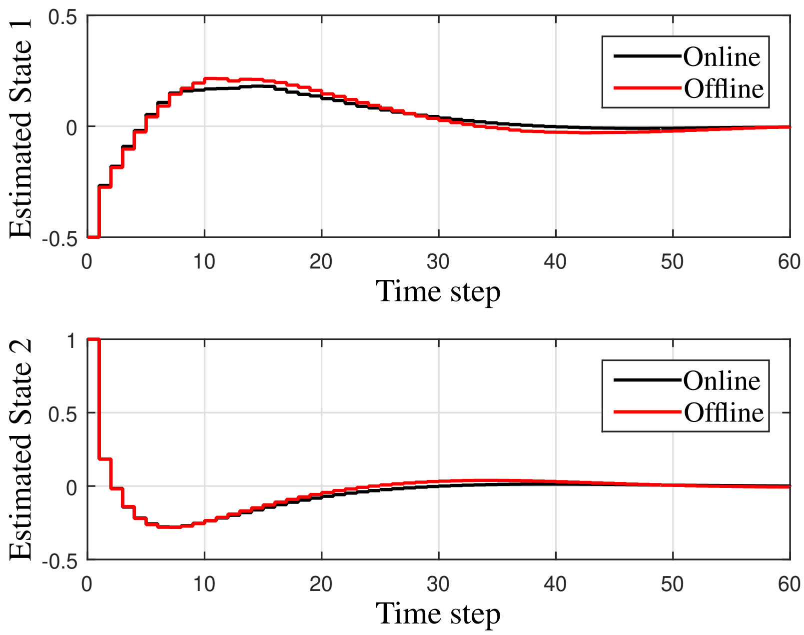
Source: The Author (2020)

Figure 23 – Online × Offline states.



Source: The Author (2020)

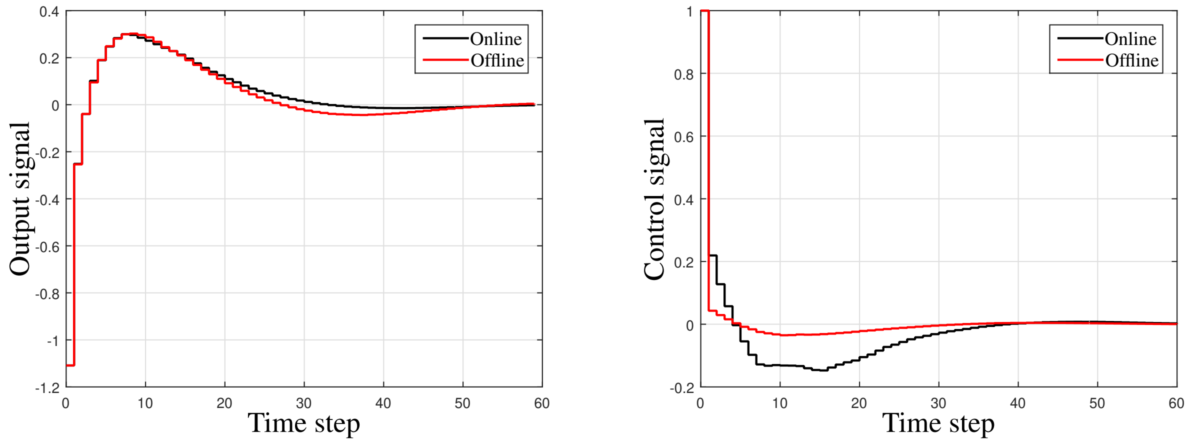
Figure 24 – Online × Offline estimated states.



Source: The Author (2020)

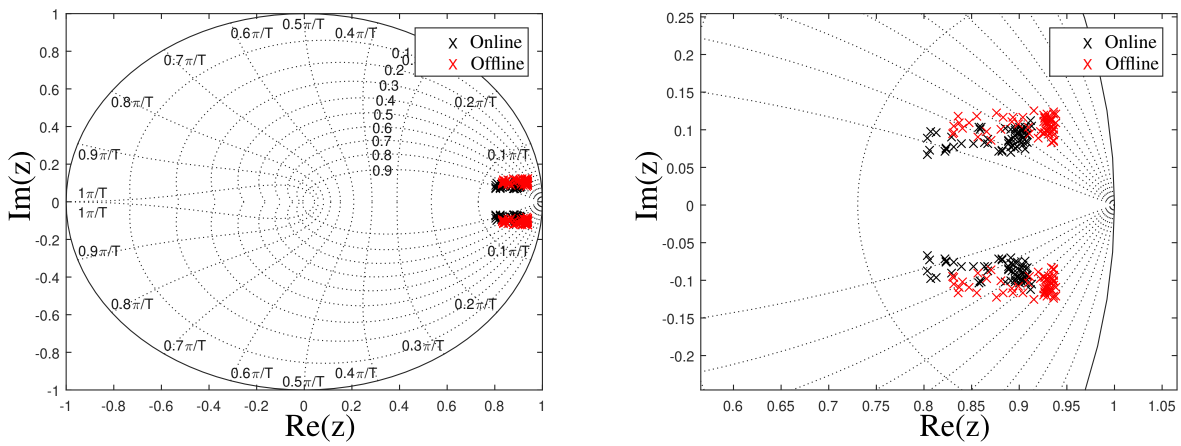
The aforementioned obtained results make explicit the viability of applying an offline output feedback FMPC for the studied model. Furthermore, the offline approach also presents an optimization in the implementation time $(12.6527s)$ of more than half of the online application $(27.4502s)$. Thus, overcoming common problems in applications with advanced control

Figure 25 – Online × Offline output and control signals.
(a) Output Signal $y(k)$ (b) Control Signal $u(k)$



Source: The Author (2020)

Figure 26 – Online × Offline closed-loop poles in the z-plane
(a) Poles allocation (b) Approximated view



Source: The Author (2020)

techniques: the high computational cost and time demand.

5.4 Chapter's Summary

The application of the proposed method to a numerical example was displayed in this chapter, aiming to investigate the viability of the studied controller and allowing the follow-up of the dissertation. Hence, a LTV state-space model was designed using the TS fuzzy methodology and then was applied to a control system designed in the block diagram from Figure 11. The computational obtained results was discussed in terms of the online and offline procedure, considering time response, poles allocation, performance indexes and stability invariant ellipsoid, all of these parameters proved the good performance of this study. It is also worth noticing the practical applicability for the offline approach, which solves a commonly found problem for advance control real applications.

6 3SSC BOOST CONVERTER

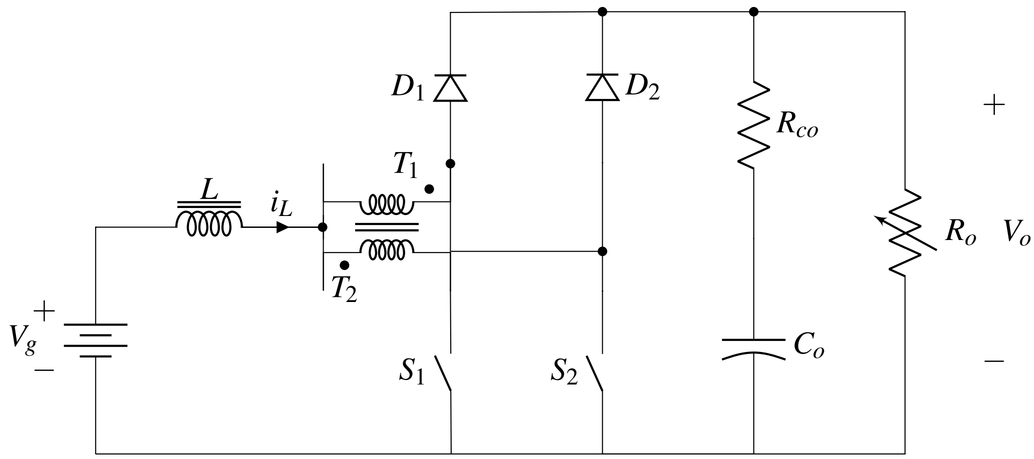
The good performance presented by the proposed output feedback FMPC for both online and offline approach in Chapter 5, allowed the progress of this study for an application in a power electronics structure, which is the main objective of this dissertation. In this scenario, the presented chapter presents the application of the proposed control procedures to a power converter with a three states switching cell (3SSC).

Hence, this chapter is composed by the following sections: Section 6.1 describes the model for the 3SSC converter, Section 6.2 presents the configuration of the proposed procedure including the control system block diagram. Next, Section 6.3 illustrates and discusses the main obtained results, and finally Section 6.4 brings the main contributions of the chapter.

6.1 3SSC Boost Converter

The application proposed in this chapter is based on a boost converter, which is a DC-DC converter with the output voltage higher than the input voltage. More specifically, the 3SSC boost converter modeled in Costa (2017) using the state-space averaging model. This is the approximate model, so the diode voltage drop, switches resistances, transformer magnetizing current and other parasitic resistances are not considered. The three states switching cell characteristic is defined because of the converter topology which is described as follows. Furthermore, Figure 27 illustrates the studied converter (BASCOPÉ, 2001).

Figure 27 – 3SSC Boost Converter



Source: Adapted from Costa (2017)

According to Bascopé e Barbi (2000), Bascopé (2001) and Costa (2017), the three state switching cell operates according to the four different modes of the switches and diodes $S_1,\ S_2$ and $D_1,\ D_2$. Three of these modes are categorized as Continuous Conduction Mode (CCM), i.e., the instantaneous inductor current is non-zero at all points in the cycle. Only the neutral state is defined for the Discontinuous Conduction Mode (DCM). Following the topology of the converter given in Figure 27, the operation modes are defined as:

- **First state** - the switches S_1 and S_2 are conducting (on) and the diodes D_1 and D_2 are reverse biased (off).
- **Second state** - S_1 and D_2 are conducting (on) and S_2 and D_1 are blocked (off).
- **Third state** - S_1 and S_2 are blocked (off) and D_1 and D_2 are conducting (on).
- **Neutral state** - S_1, S_2, D_1 and D_2 are blocked (off).

6.1.1 Boost converter state space averaging model

The adopted model for the 3SSC boost converter follows the project developed in Costa (2017), in which the system is designed through the state space averaging method developed in Middlebrook e Cuk (1976). This method is defined by the average between the models for the operation mode of S_1 and S_2 in CCM, which are expressed in the state space equations scheme as (6.1).

$$\begin{array}{ll} \dot{\tilde{x}} = A_1(t)\tilde{x} + B_1(t)V_g(t) & \dot{\tilde{x}} = A_2(t)\tilde{x} + B_2(t)V_g(t) \\ V_o(t) = C_1(t)\tilde{x} + D_1(t)V_g(t) & V_o(t) = C_2(t)\tilde{x} + D_2(t)V_g(t) \end{array} \tag{6.1}$$

where, the state variable is $\tilde{x}(t) = \begin{bmatrix} i_L & V_c \end{bmatrix}^T$, with i_L as the inductor current and V_c the capacitor voltage.

Note that, an equivalent circuit is obtained from the classic boost, which has an electrical circuit for the conducting switch and another for the blocked switch. Therefore, the average model will result from the operation of these two circuits.

Hence, following the designed procedure developed in Costa (2017) the state matrices from (6.1) are defined as:

$$S_1 \text{ mode } (D_{cycle}): \begin{array}{ll} A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{C_o(R_{co}+R_o)} \end{bmatrix}, & B_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}, \\ C_1 = \begin{bmatrix} 0 & \dfrac{R_o}{R_{co}+R_o} \end{bmatrix}, & D_1 = 0. \end{array} \tag{6.2}$$

$$S_2 \text{ mode } (1-D_{cycle}): \quad A_2 = \begin{bmatrix} \frac{R_{co}||R_o}{L} & -\frac{R_{co}}{L(R_{co}+R_o)} \\ -\frac{R_{co}}{C_o(R_{co}+R_o)} & -\frac{1}{C_o(R_{co}+R_o)} \end{bmatrix}, \quad B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} R_{co}||R_o & \frac{R_o}{R_{co}+R_o} \end{bmatrix}, \quad D_2 = 0. \tag{6.3}$$

Thence, the state space averaging model for the converter is represented by (6.4) (COSTA, 2017; MIDDLEBROOK; CUK, 1976).

$$\begin{aligned} \dot{x} &= A_t(t)x + B_t(t)u \\ y(t) &= C_t(t)x + D_t u \end{aligned} \tag{6.4}$$

with,

$$\begin{aligned} A_t(t) &= A_1(t)D_{cycle} + A_2(t)(1-D_{cycle}) \\ B_t(t) &= ((A_1(t)-A_2(t))X + (B_1(t)-B_2(t))V_g) \\ C_t(t) &= C_1(t)D_{cycle} + C_2(t)(1-D_{cycle}) \\ D_t(t) &= ((C_1(t)-C_2(t))X \end{aligned} \tag{6.5}$$

where,

$$X = \frac{V_g(t)}{R'} \begin{bmatrix} 1 \\ (1-D_{cycle})R_o(t) \end{bmatrix} \tag{6.6}$$

Thus, the state-space matrices A_t, B_t, C_t and D_t are expressed in (6.7), (6.8), (6.9) and (6.10), respectively.

$$A_t = \begin{bmatrix} -\frac{(1-D_{cycle})(R_{co}||R_o(t))}{L} & -\frac{(1-D_{cycle})R_o(t)}{L(R_{co}+R_o(t))} \\ \frac{(1-D_{cycle})R_o(t)}{C_o(R_{co}+R_o(t))} & -\frac{1}{C_o(R_{co}+R_o(t))} \end{bmatrix} \tag{6.7}$$

$$B_t = \begin{bmatrix} \left(\frac{R_o(t)}{L}\right)\frac{(1-D_{cycle})R_o(t)+R_{co}}{(R_{co}+R_o(t))} \\ -\frac{R_o(t)}{R_{co}+R_o(t)} \end{bmatrix} \left(\frac{V_g(t)}{R'}\right) \tag{6.8}$$

$$C_t = \begin{bmatrix} (1-D_{cycle})(R_{co}||R_o(t)) & \frac{R_o(t)}{R_{co}+R_o(t)} \end{bmatrix} \tag{6.9}$$

$$D_t = -V_g \frac{R_{co}||R_o(t)}{R'}. \tag{6.10}$$

Besides, the output voltage is given by $y(t) = V_o(t)$, $u(t)$ represents the control signal, and the term R' can be defined as $R' = (1-D_{cycle})^2 R_o + D_{cycle}(1-D_{cycle})(R_{co}||R_o)$.

6.1.2 Polytopic uncertainties design

The analysis of (6.7)-(6.10) shows that the states matrices of the system are modified over time according to input voltage (V_g) and the output power required (P_o). Which can be considered as the system uncertainties, since such parameters can vary unpredictably within the designed operating limits. In addition, these variables are functions of the electrical parameters load resistance (R_o) and duty cycle (D_{cycle}), respectively, as expressed in (6.11) and (6.12) (COSTA, 2017).

$$R_o(t) = f(P_o) = \frac{V_o^2}{P_o}, \;\; P_o \in [P_{o_{min}}, P_{o_{max}}] \tag{6.11}$$

$$D_{cyccle} = f(V_g) = 1 - \frac{V_g}{V_o}, \;\; V_g \in [V_{g_{min}}, V_{g_{max}}]V \tag{6.12}$$

Therefore, the converter uncertainties are represented through a polytopic structure with four vertices, given by the operation point of the local models: $f(V_{g_{max}}, P_{o_{max}})$, $f(V_{g_{min}}, P_{o_{max}})$, $f(V_{g_{max}}, P_{o_{min}})$ and $f(V_{g_{min}}, P_{o_{min}})$.

Rewriting (6.4) according to (6.11) and (6.12) the system becomes:

$$\begin{aligned} \dot{x} &= A_t\left(V_g, P_o\right) x\left(t\right) + B_t\left(V_g, P_o\right) u\left(t\right) \\ y(t) &= C_t\left(V_g, P_o\right) x\left(t\right) + D_t\left(V_g, P_o\right) u\left(t\right) \end{aligned} \tag{6.13}$$

which represents a Linear Time Variant (LTV) system. Moreover, applying Euler discretization method for a sample time T_s, (6.13) is given by (6.14).

$$\begin{aligned} x\left(k+1\right) &= A\left(V_g, P_o\right) x\left(k\right) + B\left(V_g, P_o\right) u\left(k\right) \\ y\left(k\right) &= C\left(V_g, P_o\right) x\left(k\right) + D\left(V_g, P_o\right) u\left(k\right) \end{aligned} \tag{6.14}$$

Considering the above and the electrical parameters of the 3SSC boost converter as resumed in Table 5, the discretized state matrices for the vertices of the system are given in (6.15)-(6.18).

Table 5 – Electrical parameters of the 3SSC converter

Parameter	Values
Input Voltage (V_g)	$26-36$ $[V]$
Output Voltage (V_o)	48 $[V]$
Duty Cycle (D_{cycle})	$0.25-0.46$
Switching frequency (f_s)	20.8 $[kHz]$
Sample time (T_s)	1 $[ms]$
Inductor filter (L)	35 $[\mu H]$
Output capacitor (C_o)	4000 $[\mu F]$
Capacitor intrinsic resistance (R_{co})	26.7 $[m\Omega]$
Load resistance (R_o)	$2.3-6.1$ $[\Omega]$
Output power (P_o)	$380-1000$ $[W]$

Source: The Author (2020)

$$
\begin{aligned}
&*f(36V, 1000W)\\
&A_1 = \begin{bmatrix} -0.3003 & -7.7390 \\ 0.0616 & -0.1293 \end{bmatrix} \quad B_1 = \begin{bmatrix} 541.5626 \\ 69.7156 \end{bmatrix},\\
&C_1 = \begin{bmatrix} 0.0198 & 0.9885 \end{bmatrix} \qquad D_1 = -0.7304
\end{aligned} \tag{6.15}
$$

$$
\begin{aligned}
&*f(26V, 1000W)\\
&A_2 = \begin{bmatrix} -0.0788 & -8.5609 \\ 0.0681 & 0.2528 \end{bmatrix} \quad B_2 = \begin{bmatrix} 816.3380 \\ 60.7607 \end{bmatrix},\\
&C_2 = \begin{bmatrix} 0.0143 & 0.9885 \end{bmatrix} \qquad D_2 = -1.0054
\end{aligned} \tag{6.16}
$$

$$
\begin{aligned}
&*f(36V, 380W)\\
&A_3 = \begin{bmatrix} -0.3267 & -7.9527 \\ 0.0633 & -0.1283 \end{bmatrix} \quad B_3 = \begin{bmatrix} 526.9417 \\ 71.2118 \end{bmatrix}\\
&C_3 = \begin{bmatrix} 0.01993 & 0.9956 \end{bmatrix} \qquad D_3 = -0.2802
\end{aligned} \tag{6.17}
$$

$$
\begin{aligned}
&*f(26V, 380W)\\
&A_4 = \begin{bmatrix} -0.0587 & -8.8456 \\ 0.0704 & -0.2734 \end{bmatrix} \quad B_4 = \begin{bmatrix} 806.3468 \\ 62.2455 \end{bmatrix}\\
&C_4 = \begin{bmatrix} 0.0144 & 0.9956 \end{bmatrix} \qquad D_4 = -0.3871
\end{aligned} \tag{6.18}
$$

Furthermore, the TS fuzzy representation of the 3SSC boost converter is made through a two-rules MFs implementation. Using the the duty cycle as input variable, which is a function of the input voltage, as expressed by (6.12). The fuzzy layout of this variable is done through trapezoidal membership functions, illustrated in Figure 28.

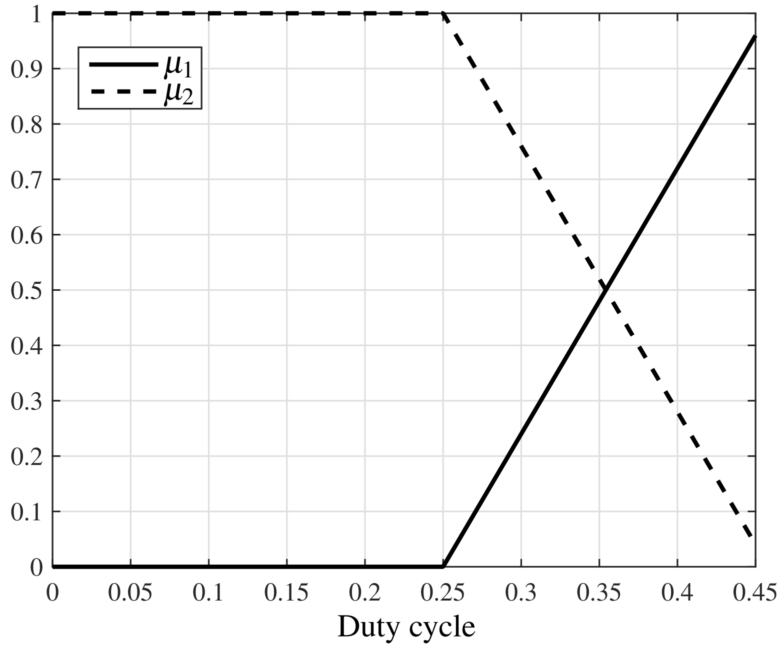


Figure 28 – Membership Functions.

6.2 Controller setup

The proposed controller procedure for the 3SSC converter application is illustrated by the block diagram in Figure 29, and mathematically is expressed as follows. As presented in Costa (2017), a integral control with two degree-of-freedom is added to the proposed scheme, with the purpose of implementing a reference tracking mechanism instead of a regulator, and also to minimize the steady-state error.

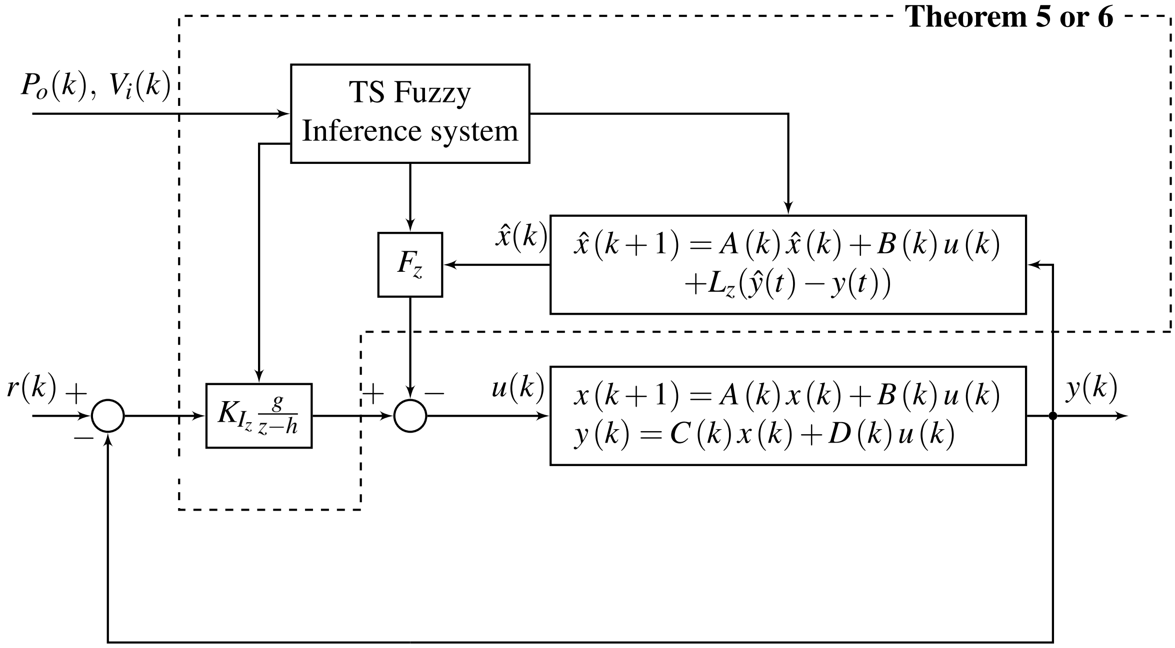
This integral mechanism is adjusted by the variables g and h, which are defined strategically in order to guarantee the controller best performance. Hence, the model expressions for the system are given by:

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{6.19}$$

And the estimated states are given as,

$$\hat{x}(k+1) = A(k)\hat{x}(k) + B(k)u(k) + L_z(\hat{y}(k) - y(k)) \tag{6.20}$$

Figure 29 – Block Diagram for the 3SSC converter



Source: The Author (2020)

The derivative of integral action is given by,

$$v(k+1) = gv(k) + h(r(k) - y(k)) \tag{6.21}$$

The control law follows the PDC rule as in,

$$u(k) = -F_z\hat{x}(k) + K_{I_z}v(k) \tag{6.22}$$

with, L_z, F_z and K_{I_z} obtained by the TS fuzzy association of the gains L_j, F_j and K_{I_j}, respectively.

Considering that the output and the estimated output are,

$$\begin{aligned} y(k) &= C(k)x(k) + D(k)u(k) \\ \hat{y}(k) &= C(k)\hat{x}(k) + D(k)u(k) \end{aligned} \tag{6.23}$$

the augmented state-equations model for this application is given in (6.24).

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} A(k) & -B(k)F_z & B(k)K_{I_z} \\ -L_zC(k) & A(k) - B(k)F_z + L_zC(k) & B(k)K_{I_z} \\ -hC(k) & hD(k)F_z & g - hD(k)K_{I_z} \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} r(k) \tag{6.24}$$

Furthermore, the weight matrices settings of the controller are defined as (6.25).

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R = 1 \tag{6.25}$$

and the variables $g = 1$ and $h = 10$ are defined by the tuning method in order to achieve the best performance for the controller. The numerical implementation of the converter follows (6.15)-(6.18), besides the initial states for real and observed states are defined as $x = [38.4615 \;\; 26]^T$ and $\hat{x} = [30 \;\; 20]^T$, respectively. The output voltage is set as $V_o = 48V$, and the input constraint is imposed as $u_{max} = 0.5$. Thence, the obtained fuzzy gains for the offline state observer are given in (6.26).

$$\begin{aligned} L_1 &= [7.9225 \quad 0.1122]^T \\ L_2 &= [8.6571 \;\; -0.2800]^T \end{aligned} \tag{6.26}$$

6.3 Simulation results

This section deals with the presentation and discussion of the results obtained through the application of the proposed output feedback FMPC to the converter 3SSC boost converter. As with Chapter 5, the responses over time and performance indexes are used to perform the comparison. Besides, the robustness of the system is illustrated by stability invariant ellipsoids.

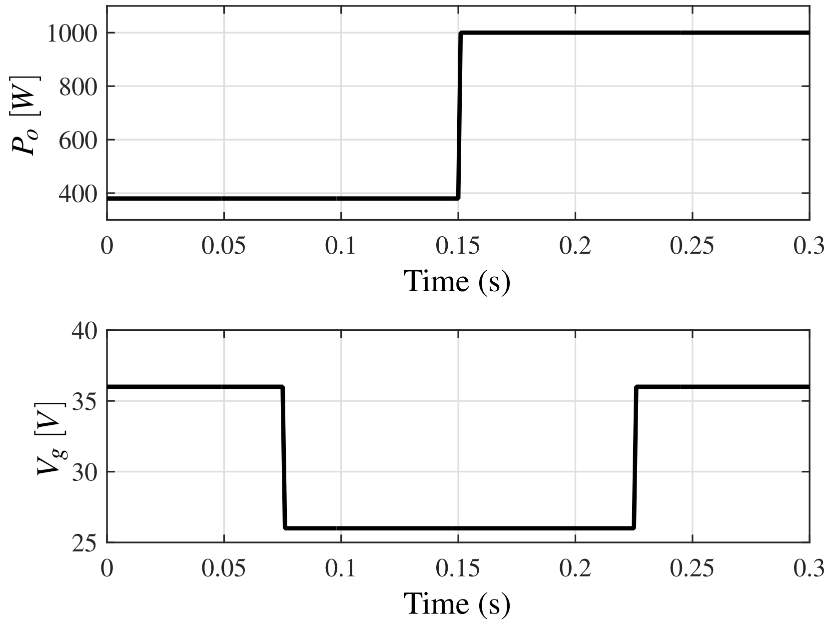
The obtained results are divided into the online and the offline procedure, presented in Sections 6.3.1 and 6.3.2. These results were obtained through numerical simulation using the YALMIP and the SEDUMI solver, to implement the LMIs. And the simulation of the 3SSC boost converter model is done using the Runge-Kutta numerical method of order 4.

6.3.1 Online approach for 3SSC boost converter

The proposed application performance is analysed in comparison with the relaxed output feedback MPC proposed in Rego (2019), adjusted to achieve its best results. Furthermore, in order to analyze the controller performance considering the limiting situations found in literature, such as change of the operating point, constraints to the process, non-linearities and non-minimum phase, the simulations are made with variation in the operation point over time, as illustrated by Figure 30.

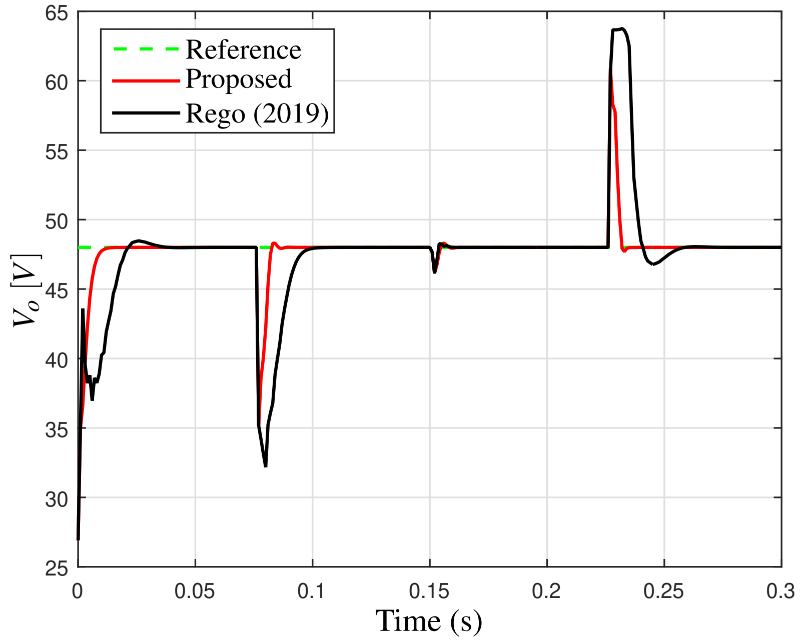
Considering this, Figure 31 displays the output response for a reference voltage $V_o = 48V$. From this figure, it can be seen that both controllers are able to maintain the reference tracking throughout the simulation time, with oscillations only in the moments of change in the operating point. However, the proposed controller presents a more stable, faster response and with lower values of overshoot ($26,8\%$) and undershoot ($26,7\%$), which indicates a better performance of this controller compared to $32,8\%$ and 33%, respectively, from Rego (2019).

Figure 30 – Operation point over time



Source: The Author (2020)

Figure 31 – Output response $y(k)$=$V_o(k)$.

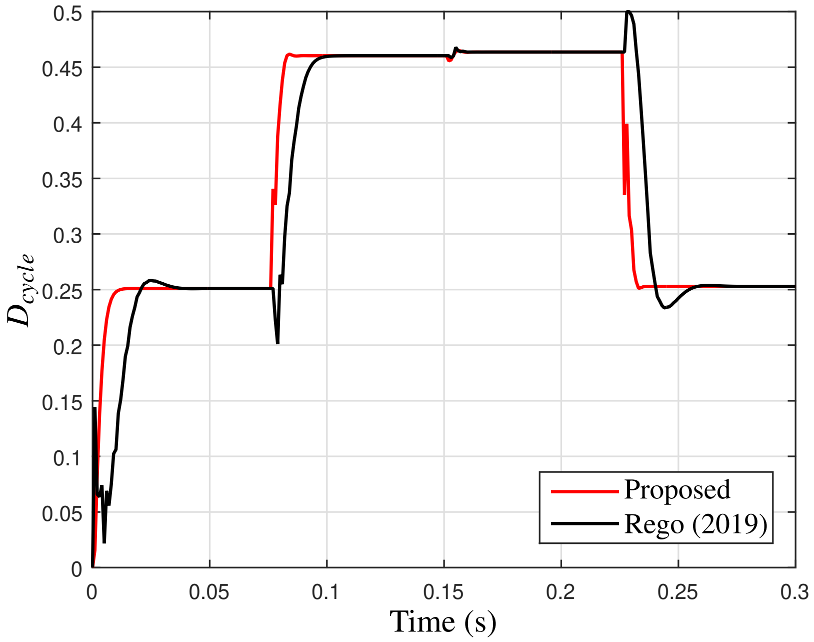


Source: The Author (2020)

The control efforts to achieve this output signal are illustrated in Figure 32, which shows that the imposed constraint is satisfied by the two controllers. Nevertheless, the control signal for the output feedback MPC from Rego (2019) presented a worse result, with more and larger oscillations and bigger overshoots and undershoot values. Furthermore, differently from Rego

(2019) the proposed technique does not reach the maximum input signal and does not present a drop in the signal.

Figure 32 – Control signal $u(k)$.



Source: The Author (2020)

Considering the aforementioned results, the performance indices for the studied controllers are listed in Table 6. By the results analysis, it is possible to affirm that the proposed controller has a better performance in terms of all evaluated metrics. It is also worth noting that besides performing better, the proposed procedure presents a high improvement of simulation time. Since, while the procedure of Rego (2019) has a simulation time of 2323s, the proposed output feedback FMPC takes 359.7s to implement.

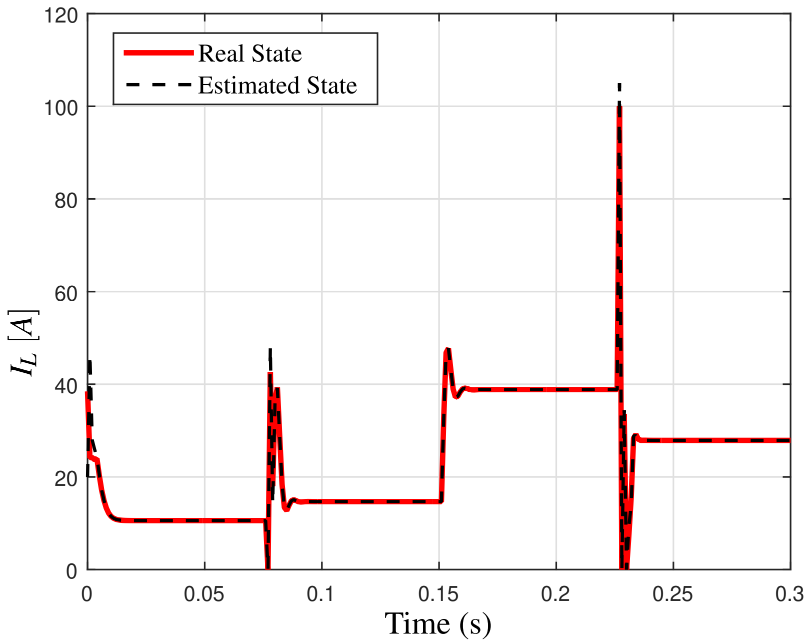
Table 6 – Performance indexes for the 3SSC boost application

	Proposed	Rego (2019)
IAE	154.1054	464.2698
ISE	1.6277×10^3	5.2553×10^3
ITAE	13.3235	52.5093
ITSE	120.6407	645.2785
J_∞	9.0326×10^5	9.5862×10^5

Source: The Author (2020)

Following with the results analysis, Figures 33 and 34 compared the real and estimated states for the proposed approach. The obtained curves prove the good performance of the fuzzy state observer in estimating the inductor current (x_1) and the capacitor voltage (x_2).

Figure 33 – Real $\times$ Estimated state x_1 for the proposed controller



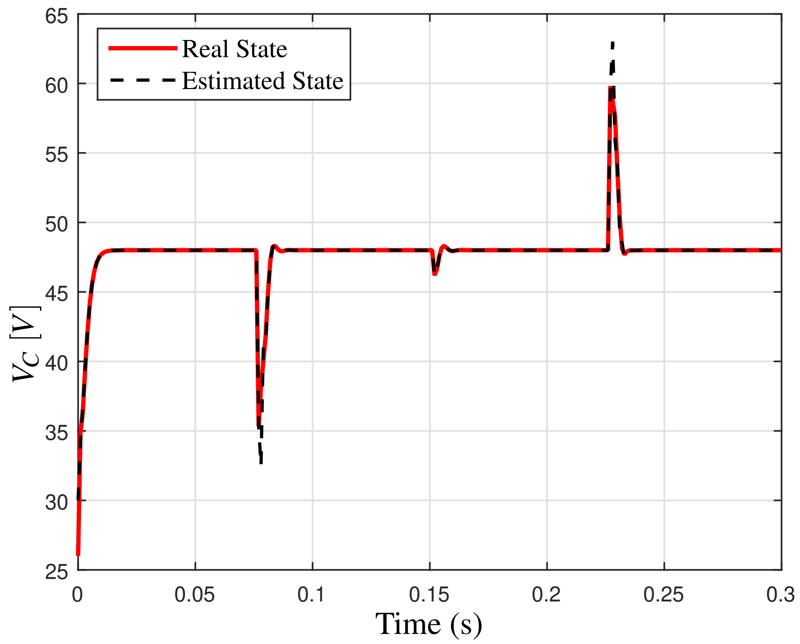
Source: The Author (2020)

6.3.2 Offline approach for 3SSC boost converter

Analogously to Section 5.3.2, an offline application of the output feedback FMPC is proposed to the 3SSC boost converter. With the results analysis also evaluated in terms of the stability invariant ellipsoids, the time response and performances indexes in comparison with the online procedure. Hence, following the steps from Theorem 6, the for the 3SSC converter, the lookup table stores the matrices Q, obtained from (4.5)-(4.8), considering a set of twenty voltage points x_{set}, which are achieved from the system states. Furthermore, the fuzzy gains for the controller with its associated points x_{set} are also kept in the lookup table.

Thus, the set of matrices Q_k, with $k = 1,\ 2,\ldots,20$ are obtained applying the offline proposed procedure for the 3SSC converter model, with the results illustrated in Figure 35, and their respective gains F_{jk} for the points $x_{set}(k)$ are listed in Table 3. Note that, in Figure 35 considering the dimension of the proposed application, the geometric representation of these matrices are given by an 3-D ellipsoid, besides the 2-D projections for these ellipsoids are also displayed. In addition, Figure 36 presents the geometric projection of those ellipsoids over time and in contrast with x_{set}. Analysing Figures 35 and 36 it is possible to see that the size of the

Figure 34 – Real $\times$ Estimated state x_2 for the proposed controller



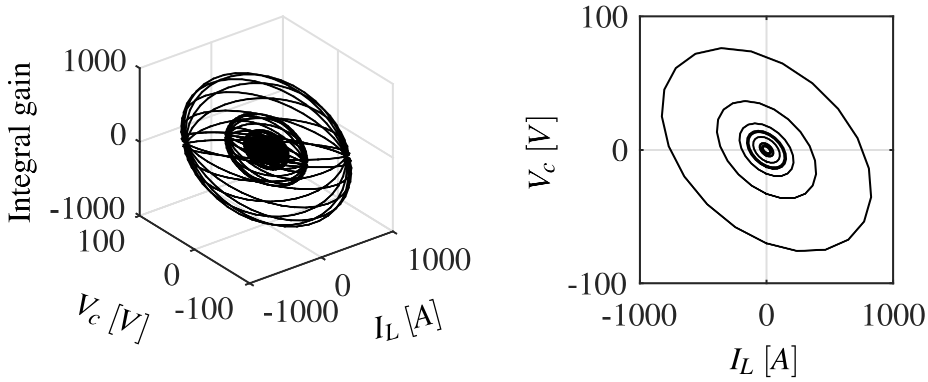
Source: The Author (2020)

ellipsoids decreases as k reaches 20, thus inferring the tendency to stabilize the ellipsoids.

Following Costa (2017), another way to confirm the system stability, using the stability invariant ellipsoids approach, is to impose the impulse response for the nominal operation point of the model ((6.15)). Besides, this response must stay within the limits of the ellipsoid, and tend to zero in a steady state. Moreover, considering the already illustrated stability tendency of the ellipsoid Q_k, the closed loop system is proven to stable for any value of k. Thus, it's up to the controller designer to choose the implemented gain. Therefore, choosing the gain for the last iteration $k = 20$, the 3-D and 2-D geometric projection of Q_{20} in contrast with the impulse response are illustrated in Figure 37. It is possible to notice that the impulse response is restricted inside the limits of the ellipsoid, and the impulse response converge to the origin, thus proving that the proposed controller guarantees the system stability.

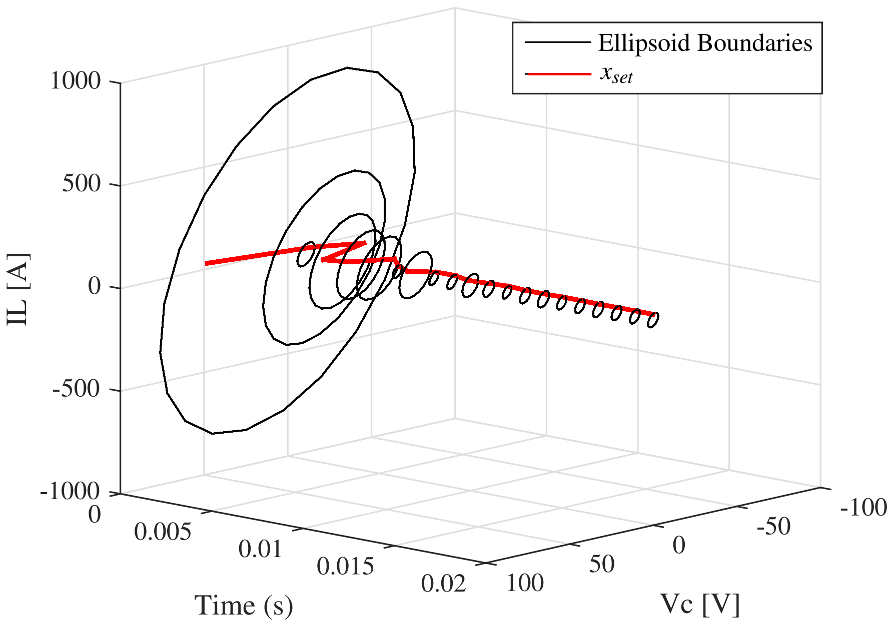
Considering, the stability characteristic proven by the invariant ellipsoid analysis, the chosen gain for $k = 20$ is implemented in order to perform a reference tracking controller as discussed in Section 6.3.1, also considering the operation point change in time given in Figure 30. The obtained results for this case are illustrated in Figures 38-39, which show the comparison between the proposed online and offline output feedback FMPC approaches. Analysing these figures, it is possible to see the ability of the offline method to stabilize the system over time and maintain the reference tracking, without significant performance loss compared to the

Figure 35 – 3-D Stability invariant ellipsoids Q_k and their 2-D projections.



Source: The Author (2020)

Figure 36 – Ellipsoids over time $\times$ x_{set}



Source: The Author (2020)

Table 7 – Set of voltage points and fuzzy gains for the offline procedure

k	$\mathbf{x_{set}}$	F_1	F_2
1	48.0000	$[-0.0005\ \ -0.0019\ \ 0.0005] \times 10^0$	$[-0.0005\ \ -0.0023\ \ 0.0005] \times 10^0$
2	-3.6942	$[-0.6727\ \ -0.7562\ \ 0.5428] \times 10^{-3}$	$[-0.0007\ \ -0.0020\ \ 0.0006] \times 10^0$
3	-23.6553	$[-0.6556\ \ -0.9611\ \ 0.5446] \times 10^{-3}$	$[-0.0006\ \ -0.0022\ \ 0.0006] \times 10^0$
4	11.3360	$[-0.0007\ \ -0.0013\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0024\ \ 0.0006] \times 10^0$
5	7.8756	$[-0.0006\ \ -0.0013\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0013\ \ 0.0006] \times 10^0$
6	-9.0638	$[-0.6605\ \ -0.7872\ \ 0.5396] \times 10^{-3}$	$[-0.0007\ \ -0.0021\ \ 0.0006] \times 10^0$
7	-0.5195	$[-0.0006\ \ -0.0013\ \ 0.0005] \times 10^0$	$[-0.0006\ \ -0.0018\ \ 0.0005] \times 10^0$
8	4.9506	$[-0.0006\ \ -0.0015\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0026\ \ 0.0006] \times 10^0$
9	-1.6958	$[-0.0007\ \ -0.0011\ \ 0.0005] \times 10^0$	$[-0.0007\ \ -0.0017\ \ 0.0006] \times 10^0$
10	-1.9171	$[-0.0007\ \ -0.0011\ \ 0.0006] \times 10^0$	$[-0.0007\ \ -0.0019\ \ 0.0006] \times 10^0$
11	1.6496	$[-0.0006\ \ -0.0019\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0029\ \ 0.0006] \times 10^0$
12	0.3478	$[-0.0006\ \ -0.0019\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0028\ \ 0.0006] \times 10^0$
13	-1.0017	$[-0.0006\ \ -0.0017\ \ 0.0006] \times 10^0$	$[-0.0007\ \ -0.0024\ \ 0.0006] \times 10^0$
14	0.2161	$[-0.0006\ \ -0.0019\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0027\ \ 0.0006] \times 10^0$
15	0.4384	$[-0.0006\ \ -0.0020\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0029\ \ 0.0006] \times 10^0$
16	-0.2874	$[-0.0006\ \ -0.0014\ \ 0.0005] \times 10^0$	$[-0.0006\ \ -0.0021\ \ 0.0005] \times 10^0$
17	-0.1152	$[-0.0006\ \ -0.0015\ \ 0.0005] \times 10^0$	$[-0.0006\ \ -0.0022\ \ 0.0005] \times 10^0$
18	0.1962	$[-0.0006\ \ -0.0019\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0027\ \ 0.0006] \times 10^0$
19	-0.0177	$[-0.0006\ \ -0.0016\ \ 0.0006] \times 10^0$	$[-0.0006\ \ -0.0024\ \ 0.0005] \times 10^0$
20	-0.0957	$[-0.0006\ \ -0.0015\ \ 0.0005] \times 10^0$	$[-0.0006\ \ -0.0025\ \ 0.0006] \times 10^0$

Source: The Author (2020)

online approach, for all evaluated parameters. Furthermore, the proposed offline procedure was also able to conduct the imposed input constraint. This conclusion is also validated by the obtained performance indices (presented in Section 5.3.1) comparison between the online and offline approaches, which are given in Table 8.

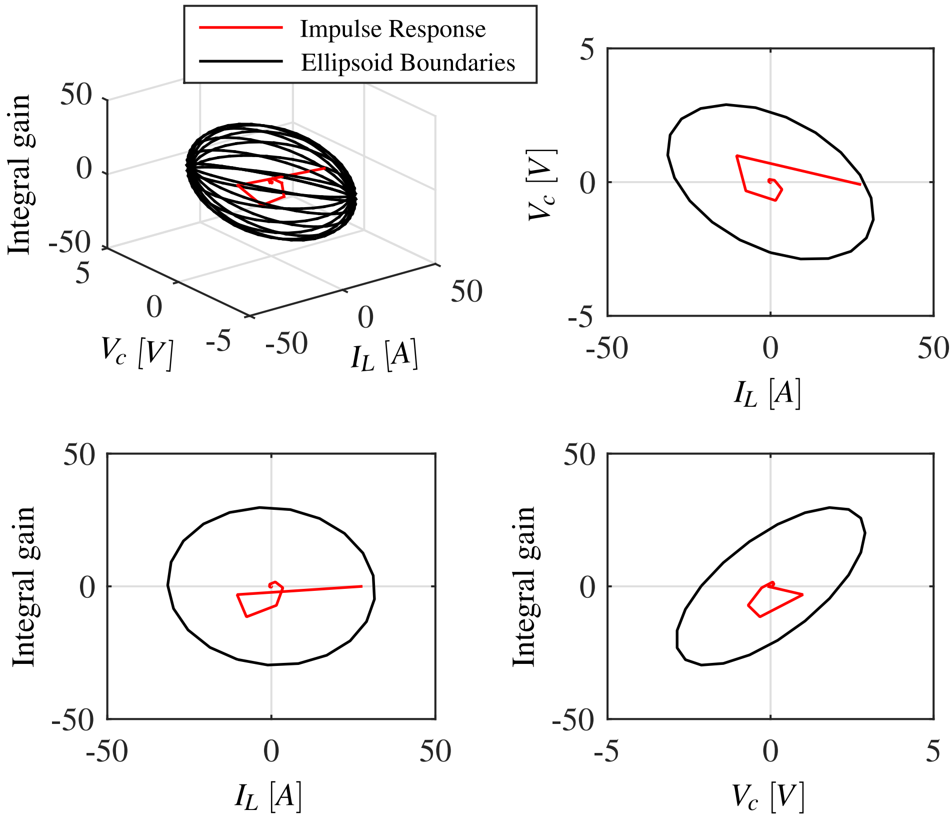
Table 8 – Online × Offline performance indexes for the 3SSC boost application

	Online	Offline
IAE	154.1054	156.7138
ISE	1.6277×10^3	1.5735×10^3
ITAE	13.3235	13.5263
ITSE	120.6407	111.8618
J_∞	9.0326×10^5	9.0360×10^5

Source: The Author (2020)

The ability of the offline approach to estimate the converter states are also evaluated, and the obtained results are displayed in Figures 40 and 41 showing that as well as with the online procedure, the fuzzy state observer presents a good performance for both inductor current (x_1) and capacitor voltage (x_2).

Figure 37 – Invariant ellipsoid × impulse response for 3SSC converter .



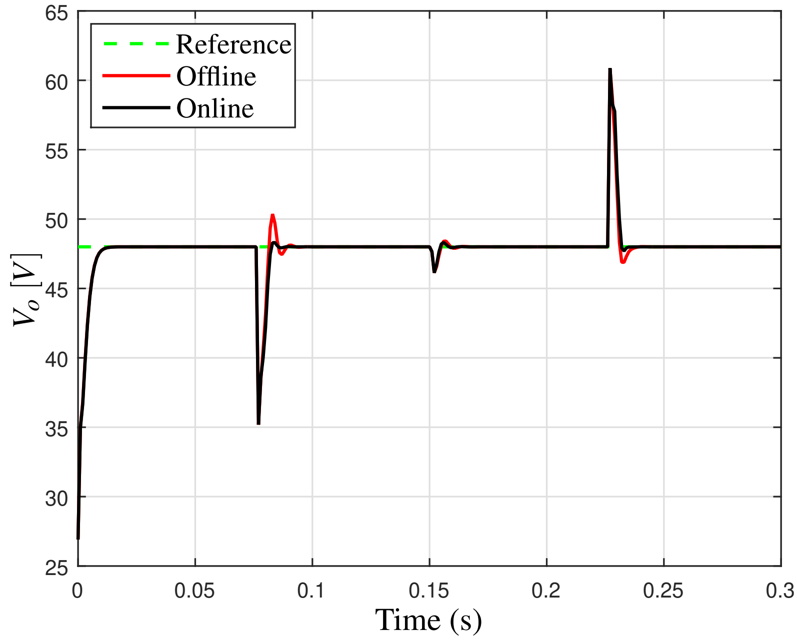
Source: The Author (2020)

When dealing with offline procedures, another important performance parameter is the reduction in the implementation wasted time, for the proposed studied the offline approach presented a simulation time of $26.8213s$ while for the online procedure this value is of $308.6427s$, representing a time gain of over ten times. Therefore, considering all obtained results it is possible to establish the viability of applying the offline output feedback FMPC for the 3SSC boost converter.

6.4 Chapter's Summary

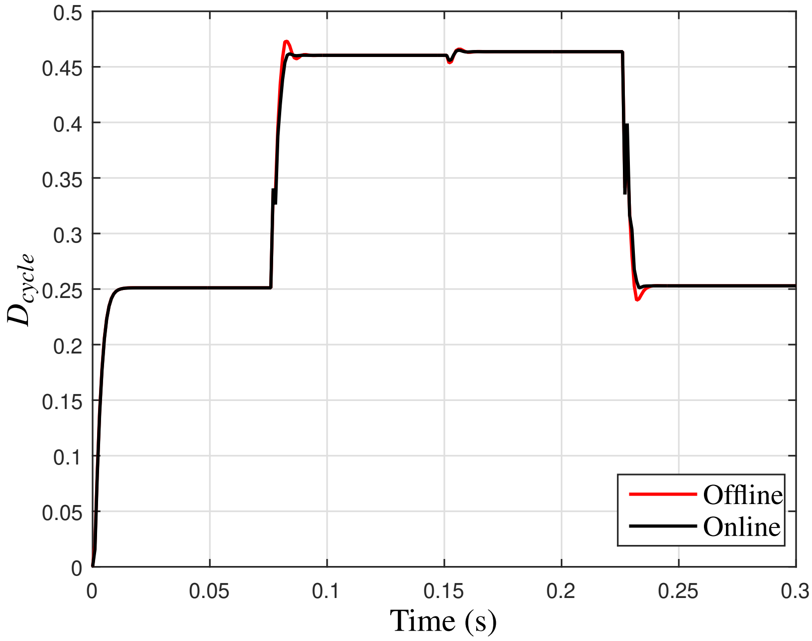
This chapter discusses the main objective of the dissertation: the application of the proposed method to the 3SSC boost converter. In this scenario, the chapter presents the mathematical design of the converter along with the controller setup, including a block diagram representation of the system. Following, the simulated results are described and analysed, first for the online method and then for the offline procedure. For the online procedure, the studied performance parameters were illustrated in comparison with the relaxed output feedback FMPC from Rego (2019), which showed the better performance of the proposed technique including

Figure 38 – Online × Offline output signal.



Source: The Author (2020)

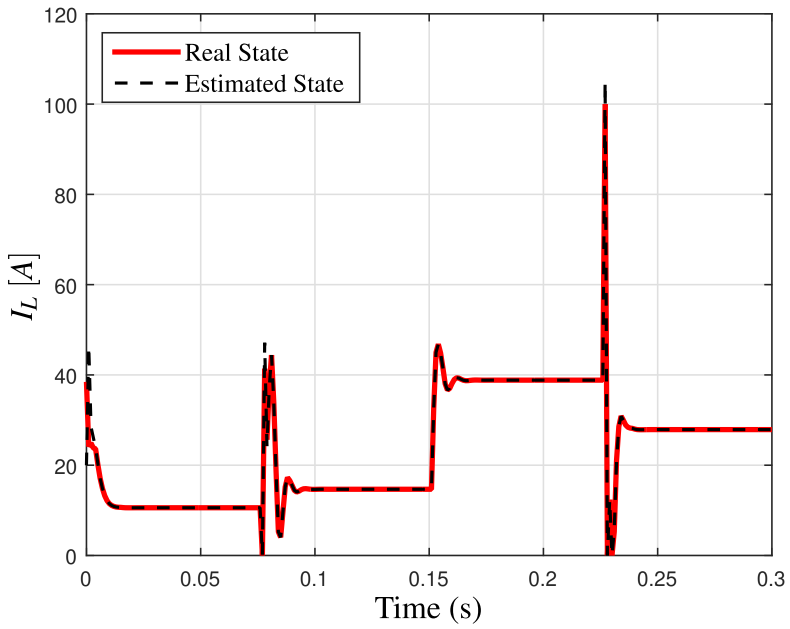
Figure 39 – Online × Offline control signal.



Source: The Author (2020)

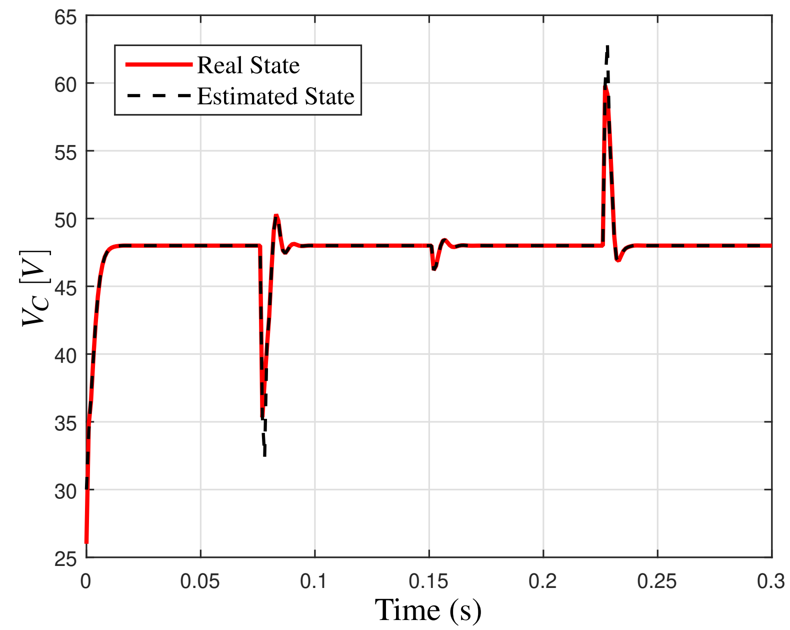
a expressive gain in the computation time waste. Besides, the offline approach also presented viable results, since the displayed results were very similar to the one obtained for the online procedure, a relevant gain in the computation time waste was also possible for this application.

Figure 40 – Real × Estimated state x_1 for the proposed offline approach



Source: The Author (2020)

Figure 41 – Real × Estimated state x_2 for the proposed offline approach



Source: The Author (2020)

7 CONCLUSION

This chapter resumes the main conclusions obtained during the study, therefore, the main findings and considerations are discussed in Section 7.1 and some proposal for the continuation of this work are presented in Section 7.2.

7.1 Final Considerations

This dissertation presented the basic theoretical aspect of fuzzy control and model predictive control, with the purpose of addressing a control method which unites the two strategies. Furthermore, the proposed control methodology was designed using new conditions to redefined an existing FMPC state feedback control, and also a fuzzy state observer. The controller-observer structure was used to perform an output feedback controller, which had its stability ensured from the development of new stability criteria. This configuration is often used to solve practical difficulties of measuring all system states.

It is also worth mentioning the realization of descriptions for the overall procedures in the form of theorems, which was defined for the online and offline approach. The latter with the objective of solving a common problem in advanced control applications: the high computational and time expense. Thus, resuming the aforementioned step the dissertation proposed an observer-based output feedback controller, considering the TS fuzzy model and the PDC control law.

Considering the exposed, the proposal was validated through two different applications. For both of them the online and offline approaches were evaluated. The online proposed theorem was analysed in comparison with benchmark output feedback MPC controllers, in order to defined the performance quality of the proposed method. And the offline approach was compared to the online procedure, with the objective of investigate the feasibility of using this technique.

The first application was developed to solve the numerical example defined in Park *et al.* (2011), and the online approach analysis was performed in comparison with the technique developed by Kim e Lee (2017) and Rego (2019). The obtained results evidenced an enhanced performance than the benchmark controllers, considering all evaluated parameters, which consist of time analysis, closed-loop poles in the z-plane and performances indexes. Now, for the offline approach the same parameters was studied and a comparison between the obtained results from the online and offline methodology was applied. Analysing both performances, it is possible to affirm the offline procedure viability, since there was a proximity of the two responses for all parameters evaluated. Besides, the invariant ellipsoid concept was used to perform the stability analysis of the offline method, confirming the robust stability of the system.

The promising results obtained in the first application allowed the sequence of the study, now considering the boost 3SSC converter, from Costa (2017), as the system model. The obtained results was analysed similarly to the numerical example. Besides, the online approach was compared with the online controller from Rego (2019), and once more all evaluated metrics established the superiority of the proposed method over the chosen benchmark. It is also worth highlighting the great computational and time gain of the online proposal of this dissertation compared to the benchmark. Then, following for the offline approach this gain is even bigger, and the controller was even able to maintain the conditions of the online performance. As with the numerical example, the system stability for the 3SSC boost application, was also evaluated in terms of the invariant ellipsoid, which also proved the overall controller stability.

Thus, considering the above, the good results obtained are encouraging and show the possibilities of continuing this study. Furthermore, this study solves some common problems found in the advanced control literature, especially when it comes to practical applications. As can be highlighted the necessity of measuring all system states to perform a state feedback control, which is sometimes impossible, and is solved for the observer-based output feedback approach. Besides, the offline procedure is a welcome improvement, since it solves the issue of high computational cost, which can make the system impracticable, and still maintain adequate performance.

7.2 Future work proposals

At the end of this dissertation, some proposals for following the work started here are defined as follows:

- Analyze the impact of fuzzy design on controller performance, through MFs.
- Use the type-2 fuzzy configuration in the controller design, with the objective of enhancing the controller performance. And also to analyze performance compared to the classical fuzzy approach.
- Compare the controller performance considering the PDC and non-PDC control laws.
- Include an Anti-Windup (A-W) actuator to the procedure, with the purpose of minimizing

the difference between the nominal and saturated response. Moreover, to develop the A-W law using the fuzzy PDC structure.

- Simulate the converter in software dedicated for modeling systems, as a way to validate the theoretical simulation.
- Compare the controller-observer output feedback with a model with disturbances.
- Perform practical applications for the proposed output feedback FMPC controller, considering the boost converter, and possibly other complex plants with non-linearities.

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